MATH 152 – Linear Systems
Test #2, Version A (TTh sections)

Spring, 2017
University Of British Columbia

Family Name: ___________________  Given Name: ___________________

Student ID: ___________________  Section: ___________________

Instructions

• You should have seven pages including this cover.
• There are 2 parts to the test:
  ○ Part A has 10 short questions worth 1 mark each
  ○ Part B has 3 long questions worth 5 marks each
• Although all questions in each part are worth the same, some may be more difficult than others – do the easy questions first!
• Use this booklet to answer questions.
• Return this exam with your answers.
• Please show your work. Correct intermediate steps may earn credit.
• No calculators are permitted on the test.
• No notes are permitted on the test.
• Maximum score = 25 Marks (attempt all questions)
• Maximum Time = 50 minutes.

GOOD LUCK!

<table>
<thead>
<tr>
<th>Part A</th>
<th>B1</th>
<th>B2</th>
<th>B3</th>
<th>Total</th>
</tr>
</thead>
<tbody>
<tr>
<td>Total</td>
<td>10</td>
<td>5</td>
<td>5</td>
<td>25</td>
</tr>
</tbody>
</table>
Part A - Short Answer Questions, 1 mark each

A1: Compute the determinant of the matrix
\[
\begin{bmatrix}
10 & 19 & 8 & 53 \\
0 & 1 & 19 & 2 \\
0 & 0 & \frac{1}{5} & 4 \\
0 & 0 & 0 & 7
\end{bmatrix}
\]

A2: Given that \( A = \begin{bmatrix} 1 & 0 \\ -1 & 2 \\ 1 & 1 \\ 3 \\ \end{bmatrix} \), and \( x = [2 \ 1] \), calculate \((xA)^T\).

A3: For \( A = \begin{bmatrix} 1 & 3 \\ -2 & 4 \end{bmatrix} \), find \( A^{-1} \).

A4: Which of the following are true for all \( 4 \times 4 \) matrices \( A \), with \( \det(A) = 10 \)? Circle all that apply.

(a) The reduced row echelon form of \( A \) is the \( 4 \times 4 \) identity matrix.
(b) \( A \) is invertible.
(c) The homogeneous equation \( Ax = 0 \) has infinitely many solutions.
(d) The rank of \( A \) is 10.
(e) \( A = A^T \).

A5: Consider the following lines of MATLAB code:

\[
A = [1 \ 1; \ 1 \ 1];
\text{rref}(A)
\]

What is the result of the last line above?
Questions A6-A7 concern the resistor network below.

**A6:** Write the linear equation involving the unknown loop currents and current source voltage that corresponds to summing the voltage drops around loop 1 (Kirchhoff’s second law).

**A7:** Write the linear equation that matches the loop currents to the current source.

**A8:** Suppose the matrix $A$ below satisfies $A^T = A^{-1}$. What are all possible values of $c$ and $d$?

$$A = \begin{bmatrix} 0 & 1 \\ c & d \end{bmatrix}$$
Questions A9-A10 below concern a game described as follows: You place a game piece on one of the three circles below. You roll a 6-sided die to figure out where your game piece moves. If you roll 1, you stay in your space. If you roll 2 or 3, you move your piece clockwise. If you roll 4, 5, or 6, you move your piece counterclockwise.

A9: Write a probability transition matrix $P$ for the game. Use the ordering $ABC$ for the states.

A10: Suppose you start on Circle A (before you’ve rolled the die at all). What is the probability that after two rolls, your piece is in Circle A?
Part B - Long Answer Questions, 5 marks each

B1: Match the matrices $A$ below with their inverse. One mark each.

(a) $A = \begin{bmatrix} -2 & -3 & -1 \\ -1 & -2 & -1 \\ 1 & -1 & -1 \end{bmatrix}$  
   (A) $A^{-1}$ does not exist.

(b) $A = \begin{bmatrix} 1 & 1 & 0 \\ 0 & 1 & 1 \\ 1 & 0 & 1 \end{bmatrix}$  
   (B) $A^{-1} = \begin{bmatrix} 1 & -2 & 1 \\ -2 & 3 & -1 \\ 3 & -5 & 1 \end{bmatrix}$

(c) $A = \begin{bmatrix} 1 & 1 & 0 \\ 0 & 1 & 1 \\ 1 & 2 & 1 \end{bmatrix}$  
   (C) $A^{-1} = \begin{bmatrix} 1/2 & 1/2 \\ -1/2 & 1/2 \end{bmatrix}$

(d) $A = \begin{bmatrix} 1 & 1 & 1 \\ 0 & 2 & 3 \end{bmatrix}$  
   (D) $A^{-1} = \begin{bmatrix} 1/2 & -1/2 & 1/2 \\ 1/2 & 1/2 & -1/2 \\ -1/2 & 1/2 & 1/2 \end{bmatrix}$

(e) $A = \begin{bmatrix} 1 & -1 \\ 1 & 1 \end{bmatrix}$  
   (E) $A^{-1} = \begin{bmatrix} 4/3 & -1/2 \\ 1/3 & 0 \\ -2/3 & 1/2 \end{bmatrix}$

(F) $A^{-1}$ is not in the list above.
B2: Let $T : \mathbb{R}^3 \to \mathbb{R}^3$ be a linear transformation. Given that

$$T \left( \begin{bmatrix} 1 \\ 1 \\ 1 \end{bmatrix} \right) = \begin{bmatrix} 1 \\ 0 \\ -1 \end{bmatrix}, \quad T \left( \begin{bmatrix} 0 \\ 1 \\ 1 \end{bmatrix} \right) = \begin{bmatrix} 1 \\ -1 \\ 0 \end{bmatrix}, \quad T \left( \begin{bmatrix} 1 \\ 0 \\ 1 \end{bmatrix} \right) = \begin{bmatrix} 1 \\ -1 \\ 2 \end{bmatrix},$$

(a) [1 mark] Write the vector $[1 \ 0 \ 0]$ as a linear combination of $[1 \ 1 \ 1]$ and $[0 \ 1 \ 1]$.

(b) [3] Let $B$ be the matrix representation of $T$. Find $B$.

(c) [1] Determine whether $B$ is invertible using a determinant calculation.
B3: Let $T_1 : \mathbb{R}^2 \rightarrow \mathbb{R}^2$ be the transformation that reflects a vector across the line $y = x$, and let $T_2 : \mathbb{R}^2 \rightarrow \mathbb{R}^2$ be the transformation that rotates a vector $45^\circ$ counter-clockwise.

(a) [1 mark] Write a matrix $R_1$ such that $T_1(x) = R_1 x$ for any $x \in \mathbb{R}^2$.
(b) [1] Write a matrix $R_2$ such that $T_2(x) = R_2 x$ for any $x \in \mathbb{R}^2$.
(c) [1] Suppose $T_3 = T_1 \circ T_2$. That is, $T_3$ is the transformation that takes an input vector in $\mathbb{R}^2$, rotates it counter-clockwise, then reflects it. Find a matrix $R_3$ such that $T_3(x) = R_3 x$ for any $x \in \mathbb{R}^2$.
(d) [2] Find a nonzero vector $x$ in $\mathbb{R}^2$ such that $T_1(x) = T_2(x)$, or show with a calculation that none exists.