Answers for Sample Exam #1

1. a) $2 \sec^2 x \tan x$, OR $\frac{2 \sin x}{\cos^3 x}$ OR $(2 \sin x)(\cos x)^{-3}$ OR . . .  
   b) $\arctan x$ OR $\tan^{-1} x$

2. $2^{-2} + (3)(5)(\ln 2)(2^{-2})$ (acceptable) OR $\frac{1 + 15 \ln 2}{4}$ (more attractive) OR even 2.8493019 (but why bother?)

3. $\frac{4}{3}$ (best) OR $1.33333$ (probably) 1.33333

4. a) $y = \frac{x - 1}{4} + \frac{1}{2}$ OR $y = x/4 + 1/4$, OR . . .
   b) 0.55 (best) OR $(1.2 - 1)/4 + 1/2$

5. a) 4
   b) leftward OR toward the origin
   c) decreasing

6. a) $-\frac{\sin x}{|\sin x|}$, OR $-1$ when $\sin x > 0$ and 1 when $\sin x < 0$, OR $-1$ for $2k\pi < x < (2k+1)\pi$ and 1 for $(2k+1)\pi < x < (2k+2)\pi$, where $k$ is an integer
   b) when $x$ is an integer multiple of $\pi$ (best) OR when $\sin x = 0$.

7. $6\sqrt{2}$

8. $f(z) = -10z$

9. a) Let $f(x) = x - e^{-x}$. Since $f$ is continuous and $f(0) < 0$ and $f(1) > 0$, there is a real number $r$ between 0 and 1 such that $f(r) = 0$ (Intermediate Value Theorem). So the equation $x = e^{-x}$ has at least one root. Since $f'(x) = 1 + e^{-x}$, and $1 + e^{-x} > 0$ for all $x$, the function $f$ is increasing, so $f(x) = 0$ for at most one value of $x$. Thus there is exactly one root.
   OR Sketch $y = x$ and $y = e^{-x}$, showing the point of intersection. The shapes of these curves can be taken as “known.”

   b) $x_{n+1} = x_n - \frac{x_n - e^{-x_n}}{1 + e^{-x_n}}$ (acceptable) OR $x_{n+1} = x_n - \frac{x_ne^{x_n} - 1}{e^{x_n} + 1}$ OR $x_{n+1} = \frac{x_n + 1}{e^{x_n} + 1}$
10. a) \( \left( \frac{1 - \ln x}{x^2} \right) x^{1/x} \), OR \( \left( \frac{1 - \ln x}{x^2} \right) e^{\ln x} \)

b) \( x = e \)

c) Since \( f'(x) < 0 \) when \( x > e \), the function \( f \) is decreasing from \( x = e \) on, so \( 3^{1/3} \) is larger than \( \pi^{1/\pi} \). (Simply writing that \( 3^{1/3} \) is about 1.44225 while \( \pi^{1/\pi} \) is about 1.43962 is not adequate.)

11. 2

12. a) \( f(x) \) is decreasing on the interval \( \infty < x \leq 0 \), increasing on the interval \( 0 \leq x \leq 2 \), decreasing on \( 2 \leq x < \infty \) OR \( f(x) \) is decreasing on \( (-\infty, 0] \), increasing on \( [0, 2] \), decreasing on \( [2, \infty) \); \( f(x) \) has a local minimum at \( x = 0 \), a local maximum at \( x = 2 \)

b) concave upward on the interval \( -\infty < x < 2 - \sqrt{2} \), concave downward on \( 2 - \sqrt{2} < x < 2 + \sqrt{2} \), concave upward on \( 2 + \sqrt{2} < x < \infty \);

inflection points at \( 2 \pm \sqrt{2} \)

c) \( +\infty \)

d) The (positive) \( x \)-axis is the only asymptote. The information given so far should be enough to produce a reasonable sketch. Experts will of course recognize \( k = \frac{1}{2} e^2 \) and \( f(x) = \frac{1}{2} x^2 e^{2-x} \); knowing this will allow anyone with a graphing calculator to check their answer.

13. \( \frac{\ln(4/3)}{\ln(10/9)} \) (days) after the beginning OR any equivalent expression OR \( t \) is about 27 days

14. \( 2\pi \) OR (ugh!) 6.28