Answers for Sample Exam #1

1. a) $2\sec^2 x \tan x$, OR $\dfrac{2\sin x}{\cos^3 x}$ OR $(2\sin x)(\cos x)^{-3}$ OR ...

   b) $\arctan x$ OR $\tan^{-1} x$

2. $2^{-2} + (3)(5)(\ln 2)(2^{-2})$ (acceptable) OR $\dfrac{1 + 15\ln 2}{4}$ (more attractive) OR even 2.8493019 (but why bother?)

3. $\dfrac{4}{3}$ (best) OR $1.\overline{3}$ (?) OR (probably) 1.33333

4. a) $y = \dfrac{x-1}{4} + \dfrac{1}{2}$ OR $y = x/4 + 1/4$, OR ...

   b) 0.55 (best) OR $(1.2 - 1)/4 + 1/2$

5. a) 4

   b) leftward OR toward the origin

   c) decreasing

6. a) $-\dfrac{\sin x}{|\sin x|}$, OR -1 when $\sin x > 0$ and 1 when $\sin x < 0$, OR -1 for $2k\pi < x < (2k+1)\pi$ and 1 for $(2k+1)\pi < x < (2k+2)\pi$, where k is an integer

   b) when x is an integer multiple of π (best) OR when $\sin x = 0$.

7. $6\sqrt{2}$

8. $f(z) = -10z$

9. a) Let $f(x) = x - e^{-x}$. Since f is continuous and $f(0) < 0$ and $f(1) > 0$, there is a real number r between 0 and 1 such that $f(r) = 0$ (Intermediate Value Theorem). So the equation $x = e^{-x}$ has *at least* one root. Since $f'(x) = 1 + e^{-x}$, and $1 + e^{-x} > 0$ for all x, the function f is increasing, so $f(x) = 0$ for *at most* one value of x. Thus there is *exactly* one root.

   OR Sketch $y = x$ and $y = e^{-x}$, showing the point of intersection. The shapes of these curves can be taken as "known."

   b) $x_{n+1} = x_n - \dfrac{x_n - e^{-x_n}}{1 + e^{-x_n}}$ (acceptable) OR $x_{n+1} = x_n - \dfrac{x_n e^{x_n} - 1}{e^{x_n} + 1}$ OR $x_{n+1} = \dfrac{x_n + 1}{e^{x_n} + 1}$

10. a) $\left(\frac{1-\ln x}{x^2}\right) x^{1/x}$, OR $\left(\frac{1-\ln x}{x^2}\right) e^{\frac{\ln x}{x}}$

b) $x = e$

c) Since $f'(x) < 0$ when $x > e$, the function f is decreasing from $x = e$ on, so $3^{1/3}$ is larger than $\pi^{1/\pi}$. (Simply writing that $3^{1/3}$ is about 1.44225 while $\pi^{1/\pi}$ is about 1.43962 is not adequate.)

11. 2

12. a) $f(x)$ is decreasing on the interval $\infty < x \le 0$, increasing on the interval $0 \le x \le 2$, decreasing on $2 \le x < \infty$ OR $f(x)$ is decreasing on $(-\infty, 0]$, increasing on $[0, 2]$, decreasing on $[2, \infty)$; $f(x)$ has a local minimum at $x = 0$, a local maximum at $x = 2$

b) concave upward on the interval $-\infty < x < 2 - \sqrt{2}$, concave downward on $2 - \sqrt{2} < x < 2 + \sqrt{2}$, concave upward on $2 + \sqrt{2} < x < \infty$; inflection points at $2 \pm \sqrt{2}$

c) $+\infty$

d) The (positive) x-axis is the only asymptote. The information given so far should be enough to produce a reasonable sketch. Experts will of course recognize $k = \frac{1}{2}e^2$ and $f(x) = \frac{1}{2}x^2 e^{2-x}$; knowing this will allow anyone with a graphing calculator to check their answer.

13. $10\frac{\ln(4/3)}{\ln(10/9)}$ (days) after the beginning OR any equivalent expression OR t is about 27 days

14. 2π OR (ugh!) 6.28