SFU-UBC-UNBC-UVic

CALCULUS CHALLENGE EXAMINATION

JUNE 09, 2016, 12:30 - 15:30

Name: ______________________________ (please print)

family name *given name*

Signature: ______________________________

Instructions:

1. Do not open this booklet until told to do so.
2. Write your name above in block letters and sign your exam.
3. Write your answer in the space provided below the question. If additional space is needed then use the back of the previous page. Your final answer should be simplified as far as is reasonable.
4. Make the method you are using clear in every case unless it is explicitly stated that no explanation is needed.
5. This exam has 15 questions on 15 pages (not including this cover page and the sheet of trigonometric identities). Once the exam begins please check to make sure your exam is complete.
6. **No** Books, papers, or electronic devices except a Ministry-approved calculator and the sheet of trigonometric identities attached to this examination shall be within the reach of a student during the examination.
7. **During the examination, communicating with, or deliberately exposing written papers to the view of, other examinees is forbidden.**

Question	Maximum	Score
1	8	
2	6	
3	6	
4	5	
5	7	
6	6	
7	6	
8	7	
9	7	
10	8	
11	6	
12	8	
13	7	
14	7	
15	6	
Total	100	

1. Compute the following limits.

[3] (a) $\lim_{x\to 6} \dfrac{\sqrt{x-2}-2}{\sqrt{x-6}}$

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[3] (b) $\lim_{x\to\infty} \left(1+\dfrac{a}{x}\right)^{bx}$

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[2] (c) $\lim_{t\to\infty} \dfrac{3t^2+8t-6}{t^2-1}$

2. Compute the indicated derivatives. You do not need to simplify your answers.

[2] (a) $f(x) = e^x - \frac{1}{x^2} + \sqrt[5]{x^2} - 10^3$

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[2] (b) $F(v) = \left(\dfrac{v}{v^3+1}\right)^6$

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[2] (c) $k(x) = \sin x \cos\left(x^2\right)$

[2] **3.** (a) State the definition of the derivative of a function $f(x)$.

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[3] (b) Use the **definition** above to find $f'(x)$, given that $f(x) = \sqrt{9 - x}$.

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[1] (c) State the domain of the function and the domain of its derivative for the function in part b).

[5] **4.** A bacteria population is 4000 at time $t = 0$ and its rate of growth is $1000 \cdot \ln 2 \cdot 2^t$ bacteria per hour after t hours. What is the population after one hour?

[7] **5.** Find the area of the region bounded by the curves $y = \sqrt{x-1}$ and $x - y = 1$.

[2] **6.** (a) State the definition of continuity of a function f at a number a.

[4] (b) For what values of the constant c is the function f continuous on $(-\infty, \infty)$? Justify your answer using the definition of continuity.

$$f(x) = \begin{cases} cx^2 + 2x & \text{if } x < 2 \\ x^3 - cx & \text{if } x \geq 2 \end{cases}$$

7. Newton's method can be used to find solutions of the equation $\sin x = x^2 - 2$.

[4] (a) Find the iteration formula to calculate x_{n+1} from x_n which will find solutions to the above equation.

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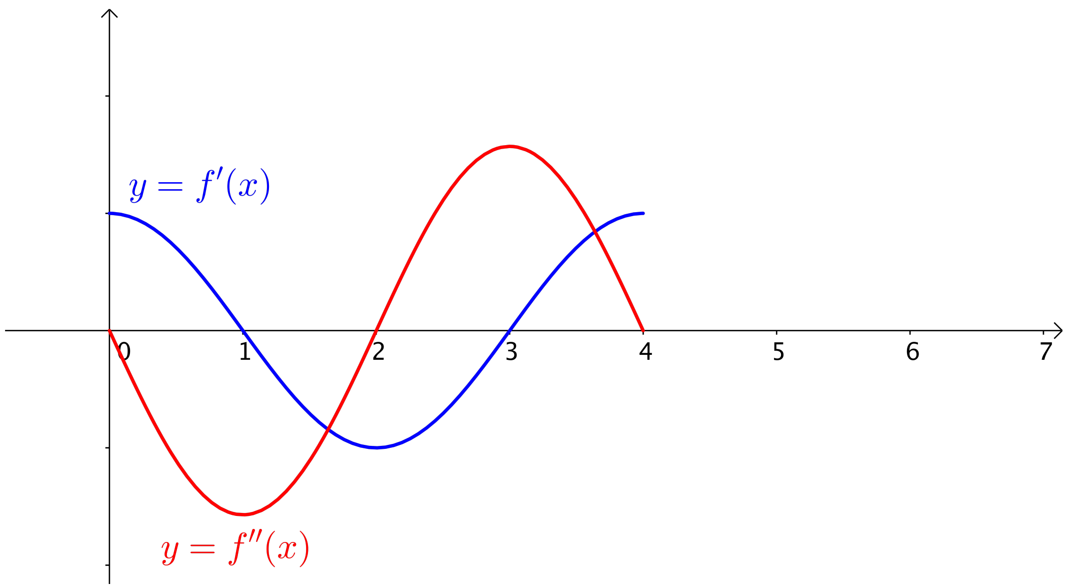
[2] (b) Perform one iteration of Newton's method to approximate the postive solution using an initial guess of $x_0 = \pi/2$.

[4] **8.** (a) Use the graphs of f' and f'' to find the intervals on which f is increasing and decreasing, and the intervals of concavity.



[3] (b) Sketch the graph of f on the figure above assuming $f(0) = 0$.

9. Suppose that we don't have a formula for $g(x)$ but we know that $g(2) = -4$ and $g'(x) = \sqrt{x^2+5}$ for all x.

[4] (a) Use a linear approximation to estimate $g(2.05)$.

[3] (b) Is your estimate in part (a) too large or too small? Explain.

10. Suppose the position of an object moving horizontally after t seconds is given by

$$s = f(t) = 2t^3 - 21t^2 + 60t, \qquad 0 \leq t \leq 6,$$

where s is measured in metres, with $s > 0$ corresponding to positions right of the origin.

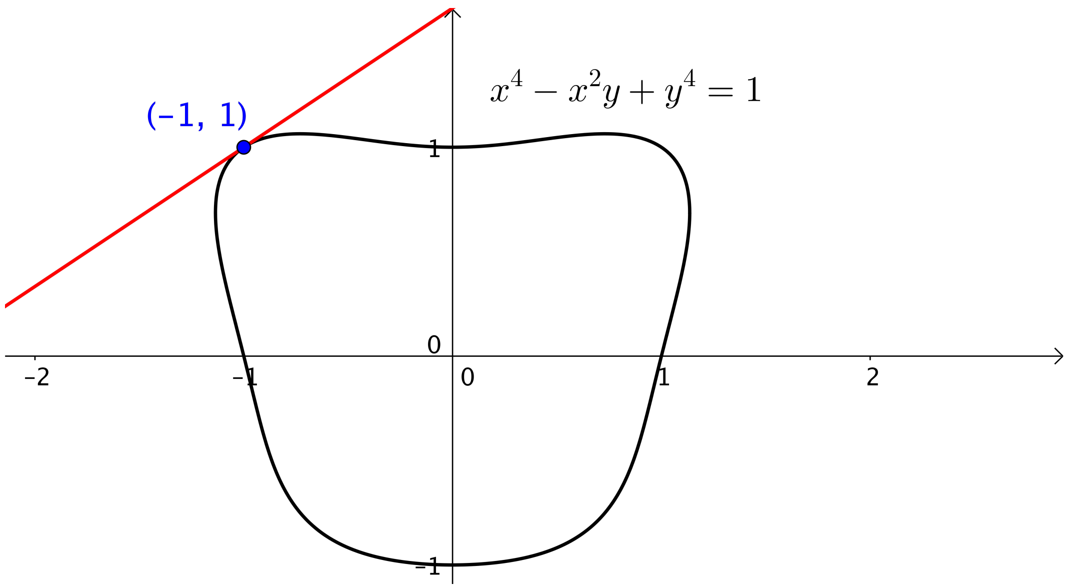
[2] (a) Find the velocity function. When is the object stationary, moving to the right, and moving to the left?

[2] (b) Determine the velocity and acceleration of the object at $t = 1$.

[1] (c) Determine the acceleration of the object when its velocity is zero.

[3] (d) On what intervals is the object speeding up? On what intervals is it slowing down?

[6] **11.** Determine an equation of the tangent line to the curve $x^4 - x^2y + y^4 = 1$ at the point $(-1, 1)$.



12. When a cold drink is taken from a refrigerator, its temperature is $5°$C. After 25 minutes in a $20°$C room its temperature has increased to $10°$C.

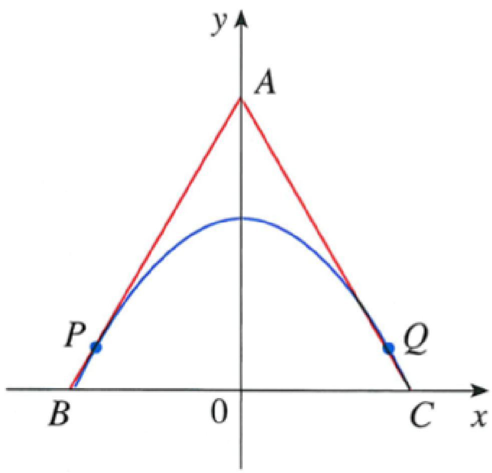
[6] (a) What is the temperature of the drink after 50 minutes?

[2] (b) When will its temperature be $15°$C?

[7] **13.** A bug is moving along the right side of the parabola $y = x^2$ at a rate such that its distance from the origin is increasing at 1 cm/min. At what rates are the x- and y-coordinates of the bug increasing when the bug is at the point $(2, 4)$?

[7] **14.** If $1200\,\text{cm}^2$ of material is available to make a rectangular box with a square base and an open top, find the largest possible volume of the box. Verify that the volume you found is maximum.

[6] **15.** Find points P and Q on the parabola $y = 1 - x^2$ so that the triangle ABC formed by the x-axis and the tangent lines at P and Q is an equliateral triangle.



Formula Sheet

Exact Values of Trigonometric Functions

θ	0	$\pi/6$	$\pi/4$	$\pi/3$	$\pi/2$	$2\pi/3$	$3\pi/4$	$5\pi/6$	π
$\sin\theta$	0	$1/2$	$\sqrt{2}/2$	$\sqrt{3}/2$	1	$\sqrt{3}/2$	$\sqrt{2}/2$	$1/2$	0
$\cos\theta$	1	$\sqrt{3}/2$	$\sqrt{2}/2$	$1/2$	0	$-1/2$	$-\sqrt{2}/2$	$-\sqrt{3}/2$	-1

Trigonometric Definitions and Identities

$\sin(-\theta) = -\sin\theta$

$\cos(-\theta) = \cos\theta$

$\sin(\theta \pm \phi) = \sin\theta\cos\phi \pm \sin\phi\cos\theta$

$\sin(2\theta) = 2\sin\theta\cos\theta$

$\cos(\theta \pm \phi) = \cos\theta\cos\phi \mp \sin\theta\sin\phi$

$\cos(2\theta) = \cos^2\theta - \sin^2\theta$

$$\sin^2\theta = \frac{1-\cos(2\theta)}{2}$$

$$\cos^2\theta = \frac{1+\cos(2\theta)}{2}$$

$\sin^2\theta + \cos^2\theta = 1$

$\tan^2\theta + 1 = \sec^2\theta$

$$\tan\theta = \frac{\sin\theta}{\cos\theta}$$

$$\sec\theta = \frac{1}{\cos\theta}$$

$$\cot\theta = \frac{\cos\theta}{\sin\theta}$$

$$\csc\theta = \frac{1}{\sin\theta}$$