UBC-SFU-UVic-UNBC
Calculus Examination
4 June 2015, 12:00-15:00PDT

Time: 3 hours

Name _____ **Signature** _____

School _____ **Candidate Number** _____

Rules governing examinations

1. Show all your work! Full marks are given only when the answer is correct, and is supported with a written derivation that is orderly, logical, and complete. Part marks are available in every question.
2. Calculators are optional, not required. Correct answers that are calculator ready, like $3 + \ln 7$ or $e^{\sqrt{2}}$ are fully acceptable.
3. Any Calculator acceptable for the Provincial Examination in Principles of Mathematics 12 may be used.
4. A basic formula sheet has been provided. No other notes, books, or aids are allowed. In particular, all calculator memories must be empty when the exam begins.
5. If you need more space to solve a problem on page n work on the back of page $n - 1$.
6. Candidates suspected of any of the following, or any other similar practices, will be immediately dismissed from the examination by the examiner/invigilator, and given a grade of zero:
   (a) speaking or communicating with other candidates, unless otherwise authorized;
   (b) purposely exposing written papers to the view of other candidates or imaging devices;
   (c) purposely viewing the written papers of other candidates;
   (d) using or having visible at the place of writing any books, papers or other memory aid devices other than those authorized by the examiner(s); and,
   (e) using or operating electronic devices including but not limited to telephones, com puters, or similar devices other than those authorized by the examiner(s)–(electronic devices other than those authorized by the examiner(s) must be completely powered down if present at the place of writing).

Question	Points	Score
1	45	
2	9	
3	8	
4	10	
5	10	
6	8	
7	10	
Total:	100	

1. (45 points) **Short Problems**. Each question is worth 3 points. Put your answer in the box provided and show your work. No credit will be given for the answer without the correct accompanying work, except for multiple choice questions.

   (a) Compute $\lim_{x\to 0} \dfrac{\sqrt{x+1}-1}{x}$.

   Answer:

   (b) Compute $\lim_{x\to \infty} \dfrac{x^2+\sin x}{(x-1)(x+1)}$.

   Answer:

   (c) Differentiate the function

   $$f(x) = (x+1)^{\cos x}$$

   .

   Answer:

(d) Let $h(x) = e^{f(x)} + [f(x)]^2$ where $f(1) = 2, f'(1) = 5$. Find $h'(1)$.

Answer:

(e) Sketch the graph of a continuous function $f(x)$ satisfying:

$$\lim_{x \to 0^-} f'(x) = 1, \lim_{x \to 0^+} f'(x) = -1, f(0) = 3$$

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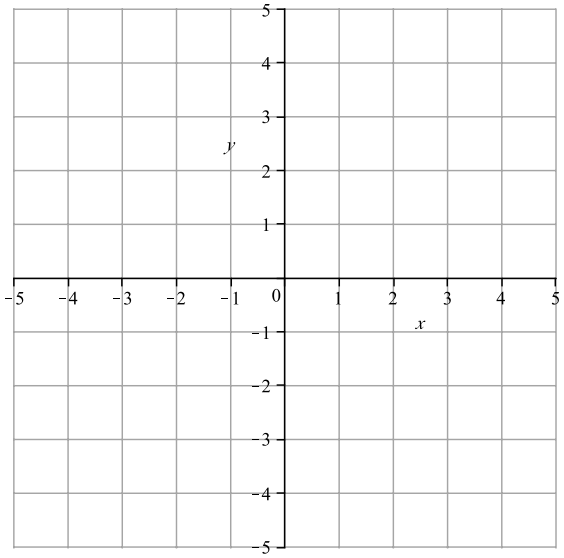

(f) Let $y(x) = e^{rx}$. For what value of r does $y(x)$ satisfy $y(x)'' - 4y(x)' + 4y(x) = 0$?

Answer:

(g) Find the equation of the tangent line to the graph $y = e^x + x^2$ at the point $(0, 1)$.

Answer:

(h) Let $L(x)$ be the linear approximation of $f(x) = \cos x$ at the point $x = 0$. Find $L(.25)$.

Answer:

(i) Evaluate $\displaystyle\int_0^4 \frac{x}{x^2+1} dx$

Answer:

(j) You invest \$ 100,000 into an account. After t years, the amount in the account satisfies $A'(t) = .07A(t)$. Your plan is to retire once the rate of growth of your investment is \$ 10,000 per year. In how many years will you retire?

Answer:

(k) Give an example of a function $f(x)$ satisfying $f(0) = 1, f'(0) = 2, f''(0) = 3$.

Answer:

(l) If $f'(a)$ exists, then $\lim_{x \to a} f(x)$

(i) equals $f(a)$.
(ii) equals $f'(a)$.
(iii) must exist, but may not equal $f(a)$ or $f'(a)$.
(iv) might not exist.
(v) None of the above.

Answer:

(m) On what intervals is the function $f(t) = t^2 e^t$ increasing? Decreasing?

The intervals of increase are:

The intervals of decrease are:

(n) Find the derivative of the function $f(x) = \ln(1 + \ln(1 + \ln x))$

Answer:

(o) Show that the equation $5^x - 10x - 7 = 0$ has a solution.

Long Problems. In questions 2 - 6, show your work. No credit will be given for the answer without the correct accompanying work.

2. Consider the function

$$f(x) = \frac{2x^3}{3x^2 - 9}.$$

Its first derivative is given by

$$f'(x) = \frac{2x^2(x^2 - 9)}{3(x^2 - 3)^2}.$$

(a) (4 points) On which intervals is $f(x)$ increasing? On which intervals is $f(x)$ decreasing?

The intervals of increase are:

The intervals of decrease are:

(b) (2 points) Find the x coordinate of all local maxima, local minima. Be sure to indicate which is which.

(c) (3 points) State (with explanation) whether $f(x)$ has any vertical or horizontal asymptotes.

3. (8 points) Sketch the graph of a function $h(x)$ which:

   (a) has vertical asymptotes at $x = -2$ and $x = 2$, and is defined everywhere else

   (b) has $h(-3) = -3$, $h(0) = 0$, $h(3) = 3$,

   (c) is increasing on $(-\infty, -3), (-2, 2), (3, \infty)$ and decreasing everywhere else on its domain.

   (d) is concave up on $(-\infty, -4), (0, 2), (2, 4)$ and concave down everywhere else on its domain.

4. (10 points) A steel company manufactures nuts and bolts. When x nuts are produced, they can be sold for $-3x + 500$ dollars each. When y bolts are produced, they can be sold for $-y + 300$ dollars each. Nuts and bolts weigh 0.5 kg each.

   (a) How much money does the company make by producing x nuts and y bolts (your answer will be in terms of x and y)?

   (b) What is the maximum amonut of money the company can make from 100 kg of steel?

5. (10 points) A spot light sits on the ground and shines on a wall 20m away. Jenny is 2m tall and stands directly between the wall and the spotlight so that the light casts a shadow of her on the wall.

   (a) Draw a diagram showing the wall, the spotlight, Jenny and her shadow on the wall.

   (b) At what rate is the height of her shadow changing if Jenny is 10m from the wall and approaching the spotlight at 1m/s?

6. (8 points) Use the limit definition of the derivative to show $\frac{d}{dx}(\frac{1}{x+1}) = -\frac{1}{(x+1)^2}$.

7. Consider the curve $x^2 + y^3 - 2xy = 0$. Assume that the point $(x, y) = (1, 1)$ lies on the curve, and that nearby points on the curve satisfy $y = f(x)$ for some function of $f(x)$.

   (a) (3 points) Use implicit differentiation to find $f'(1)$.

   (b) (4 points) Use implicit differentiation to find $f''(1)$.

(c) (3 points) Approximate $f(1.02)$ using the linear approximation of $f(x)$ at $x = 1$. State, with explanation, whether you expect this to be an overestimate, or an underestimate of $f(1.02)$.

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