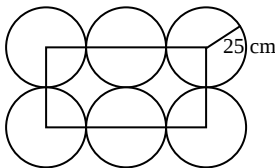


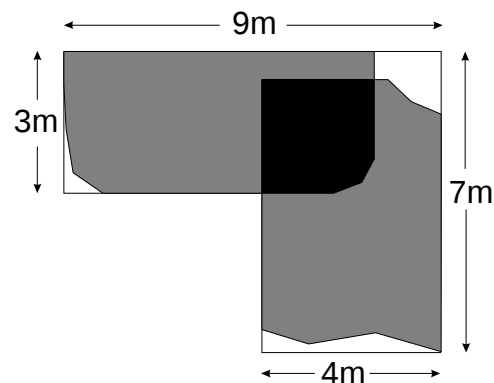
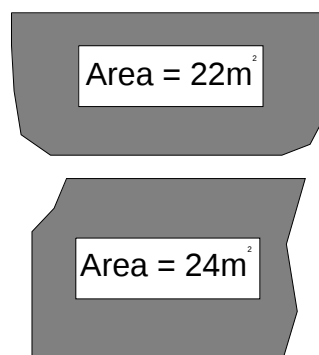
UBC Grade 6–7 Workshop Problems, 2005–2006

1. Shirley is designing her Science Fair display booth. For the display table, she uses 6 round barrels of radius 25 cm as legs, with the corners of the rectangular table top fitting in the centre of each barrel, as shown below. What is the area of the tabletop?



2. Three friends, Ali, Bali, and Clay, moved into a new home two weeks ago. Their neighbour, Mrs. Baker, brought them three homemade pies as a housewarming gift last Sunday. Ali eats $\frac{1}{6}$ of a pie a day, Bali eats $\frac{1}{2}$ of a pie a day, and Clay eats $\frac{1}{3}$ of a pie a day. On what day will Mrs. Baker have to bake more pies in order to keep her new neighbours happily fed?
3. Prof. Ecostein teaches in the Ecology Department at UBC. He wants to take the bus to work every day, and has two choices: he can walk 1 km to the stop for a local bus, which takes 20 minutes to reach UBC from the bus stop, or he can walk 2 km to the stop for an express bus, which takes 16 minutes to reach UBC. The waiting time at the bus stop for the local bus is 10 minutes, and for the express bus it is 5 minutes. Prof. Ecostein walks at a rate of 6 km/h. Should he take the local bus or the express bus?
4. Danielle has a very unusual 12-hour watch. Every time the hour hand passes three even numbers, it immediately jumps back to the previous prime number; then the watch continues until it has again passed three even numbers at which point it jumps back to the previous prime number, etc. If Danielle's watch is started at 11 a.m. today, and reads the correct time then, what is the actual time when her watch next reads 11 o'clock? After that time, when will the watch first read the correct time?
5. Rachel is looking for a house to buy. In the rectangular backyard, she wants to build two square swimming pools, each of length 5 m. There has to be a 2-m-wide sidewalk around each pool, but this sidewalk can be shared between the pools. What is the smallest backyard Rachel can consider in order to build these pools?
6. Toys“R”Us is having its Enormongo Year End Sale. No GST or PST is charged, and in addition prices are reduced as follows. Every item that has a regular price over \$100 has its sticker price reduced to $\frac{3}{4}$ the regular price, and all items are on sale for $\frac{1}{3}$ off the sticker price. Jubilee buys an electric keyboard for \$80, and Connie buys a Star Wars Saga DVD for \$40. What were the regular prices of the keyboard and the DVD?
7. Wonderwoman is 1400 km away from Metrotown, and Superman is 600 km away from Metrotown, in the opposite direction. The two superheroes fly to Metrotown at speeds of 200 km/h and 100 km/h respectively. Superfly starts on Wonderwoman and flies back and forth between the superheroes at 300 km/h, stopping when it lands on one of our heroes at Metrotown. What is the total distance Superfly travels before stopping at Metrotown?
8. Daniel and Willy race around an oval track with a circumference of 400 m. They start at the same place on the track but run in opposite directions, each at a constant rate. The winner is the first one who runs 4 km. If Willy runs at $\frac{2}{3}$ the speed of Daniel, how many times will the two runners cross paths during the race?

9. Kaili lives in a 20-storey apartment building that is 50 m tall and has a square base with length 15 m. For each storey of the building, there are four windows on each side. Each window measures 1 m high by 2 m wide. There is also a swimming pool on the roof, which is a square of length 5 m. Ivy cannot grow on glass or water, but otherwise completely covers the exterior surfaces of the building. What is the total area the ivy covers?
10. Darren's Designers is a company that wants to be able to divide its workforce as follows. It should be possible to form teams of 2 workers each with 1 worker leftover (to supervise the teams), teams of 3 workers each with 1 leftover, and teams of 5 workers each with 1 leftover. What is the smallest number of workers the company can have?
11. A googol is the number $1000\dots 0$, where there are 100 0s. A Gigantic Number is a number formed by writing down the digits 1, 2, 3, etc. For example the 11th Gigantic Number is 1,234,567,891,011, and this is the first Gigantic Number larger than one trillion (1 followed by 12 0s). What is the units digit of the first Gigantic Number that is larger than a googol?
12. Julie has 14 pairs of socks, each pair a different colour. How many different unmatched pairs can she make? Note that for example one red sock together with one blue sock is not counted as different from the matching red sock together with the matching blue sock.
13. Mr. Brown wins the \$60-million jackpot in Lotto 649. He has 10 friends with whom he shares the prize. He gives $\frac{1}{11}$ of the jackpot to his best friend, $\frac{1}{12}$ of the remaining money to his second-best friend, $\frac{1}{13}$ of what is left to his third-best friend, etc. How much money does Mr. Brown keep for himself?
14. Mrs. Williams wins the Lotto 21 jackpot. The prize is paid entirely in loonies and toonies. She gives all the money away to her grandchildren, ensuring that each child receives the same number of coins. Victoria receives $\frac{1}{3}$ of the loonies and $\frac{1}{12}$ of the toonies; Jordan receives $\frac{1}{8}$ of the loonies and $\frac{1}{2}$ of the toonies. How many grandchildren does Mrs. Williams have?
15. Four friends decide to play gin rummy for money in the following way. Each player starts with \$6. After each game of rummy, everyone except the winner puts \$1 into the pot. Each player continues playing until he or she has run out of money. The pot keeps growing until only one player is remaining, and then that player wins the entire pot. What is the maximum number of games needed to determine the winner?
16. Chris's Crummy Carpet-Installation Company is hired to carpet the room shown below right (assume all corners are right angles). The company tries to cover the floor using the two pieces of carpet shown below left. As you can see, some parts of the floor are left uncovered (shown in white) and some parts are covered by both carpets (shown in black). Find the area of the overlap if it is equal to twice the area of the uncovered parts of the floor.



SOLUTIONS

Note: These concise solutions are meant for workshop leaders and teachers. Presentations to Grade 6/7 students should include additional detail and motivation. The solutions outlined here are considered appropriate for school students at this level; alternate solutions are often possible.

1. Answer: 5000 cm^2 . The rectangle's height is 2 radii and its length is 4 radii, and multiplying 50 by 100 gives the answer. A warm-up problem.
2. Answer: Wednesday. Since $1/6 + 1/2 + 1/3 = 1$, exactly 1 pie is eaten per day. The 3 pies will be completely consumed on Tuesday, so on Wednesday Mrs. Baker needs to get baking again! Students who have difficulty doing the addition of fractions can instead be urged to break up the pies into 6 slices. Another warm-up.
3. Answer: local bus. Add up the times for the two scenarios. For the local bus, the walking time is 10 min, the waiting time 10, and the time on the bus is 20 minutes, so 40 min total; for the express bus the times are 20 min, 5 min, and 16 min respectively, or 41 min in total. Probably also a warm-up.
4. Answers: 1 a.m.; 11 p.m. two days from now (60 hours from 11 a.m. today). When the watch reaches 4 p.m., it will jump back 1 hour, so that it reads 3 o'clock even though it is actually 4 p.m. Now it continues till it reads 8 o'clock, when it is actually 9 p.m., and jumps back 1 more hour, so that it is reading 7 o'clock at 9 p.m. In 4 more hours, i.e. at 1 a.m., it will read 11 o'clock. For the second part of the problem, continue running the clock till it reads 12 o'clock, at 2 a.m. The clock then jumps back to 11 o'clock again. So, it has lost 3 hours, and we are back at our initial state in which the clock reads 11 o'clock and moves forward to reading 4 o'clock, 3 o'clock, etc. After 1 more 15-hour cycle, the clock will have lost another 3 hours and be back at its initial state again; after 2 additional cycles (60 actual hours altogether), the clock will correctly read 11 o'clock. Since the final hour lost in each 15-hour cycle is lost at the very end of the cycle, this is the earliest time at which the clock *first* reads the correct time again.
5. Answer: 9 m by 16 m. The most efficient configuration is side-by-side pools separated by 2 m; adding in a 2-m perimeter gives dimensions of $5 + 2 + 2$ and $2 + 5 + 2 + 5 + 2$ respectively. A picture is certainly necessary here.
6. Answers: \$160 and \$60. Start with the DVD. Before the final $1/3$ reduction, the price was \$60, since $1/3$ of \$60 is \$20. The DVD can't have had a regular price over \$100, since $3/4$ of \$100 is \$75, so only the $1/3$ discount applies. Now consider the keyboard. Since \$80 is twice \$40, its price before the $1/3$ discount was \$120. But since this price is over \$100, the keyboard was also subject to the $3/4$ price reduction. Now note that $3/4$ of \$160 is \$120. Students at this level will likely not appreciate an argument such as: " $x = (3/4)y$ implies $y = (4/3)x$ "; it's best to encourage them to use trial and error to determine the prediscounted prices.
7. Answer: 2000 km. The most efficient solution involves using the relative speeds of Superfly with respect to Superman and Wonderwoman of 400 km/h and 500 km/h respectively, but this notion will likely be too advanced for this grade level. Instead, consider what happens during various discrete time intervals; the numbers here are chosen so that only multiples of an hour need to be invoked. After 4 hours, Superfly is $1400 - 4 \times 300 = 200$ km from Metrotown, and Superman is $600 - 4 \times 100 = 200$ km away, on the other side. Another hour later, our fly, having flown another 300 km, is 100 km from Metrotown on the other side, and, guess what, so is Superman! So, after flying 1500 km in 5 hours, the fly is on Superman. At this time, Wonderwoman is $1400 - 5 \times 200 = 400$ km away from Metrotown. Now let another hour elapse. Wonderwoman is then $400 - 200 = 200$ km away from Metrotown, and since the fly travels 300 km in an hour, so is it! So, the second leg of the fly's travels takes an hour, and involves a distance of 300 km; we have 1800 km so far. But during this latest one-hour period, Superman has reached Metrotown, since he was only 100 km away at the beginning of this period. So, the fly just has to travel the final 200 km back to Metrotown, and the three legs of its journey total 2000 km. Draw pictures!

8. Answer: 16. Consider each of Daniel's 10 laps before he wins the race. On the first lap, he crosses Willy once, and at the end of this lap Willy is $1/3$ of a lap away from the starting point, where Daniel is, heading toward Daniel. On Daniel's second lap, he crosses Willy early in the lap, and then again later on; on Daniel's third lap he crosses Willy twice, including the crossing at the very end of the lap. So, during Daniel's first 3 laps there are a total of 5 crossings. We are back where we started (periodicity again – see question 4!) and repeat. After 9 laps there have been 15 crossings, and then there is one final crossing during Daniel's 10th lap. Alternate solution: Daniel meets Willy every $3/5$ of a lap he (Daniel) runs (explain why), and $10/(3/5)$ has integral part 16. A diagram helps here.
9. Answer: 2560 m^2 . Each side of the building has an area of $50 \times 15 = 750 \text{ m}^2$ and $4 \times 20 = 80$ windows, each of area 2 m^2 , so the ivy-covered area on each side is $750 - 160 = 590 \text{ m}^2$. That gives a total vertical ivy-covered area of $590 \times 4 = 2360 \text{ m}^2$. The top of the building has an ivy-covered area of $15 \times 15 - 5 \times 5 = 200 \text{ m}^2$; adding 2360 and 200 gives our answer. The problem can also be done by computing the total area of the 4 sides of the building and subtracting the area of all 320 windows, but the given method is probably a tad more efficient.
10. Answer: 31. The required number is the smallest positive integer that leaves a remainder of 1 when divided by each of 2, 3, and 5. Assuming we can't have 0 teams (the company wouldn't be very productive if so!), the remainder upon divisibility by 5 means the number of workers must be in the sequence 6, 11, 16,... Try these out and check that the smallest member of this sequence that leaves a remainder of 1 when divided by each of 2 and 3 is 31. Students might be asked to ponder: $31 = 1 + 2 \times 3 \times 5$.
11. Answer: 5. The 11th Gigantic Number (GN) has 13 digits. In order for a GN to be greater than a googol, it has to have at least 101 digits. In fact at least 101 digits is exactly what we need, since a googol is not a GN. Since $101 - 13 = 88$, and from the 9th GN to the 98th GN we add 2 digits to get to the next GN, the first GN larger than a googol is the $11 + 44 = 55$ th GN. This GN ends as ...5455 and has units digit 5.
12. Answer: 91. Say the first colour is red and the second is blue. There are 13 different unmatched pairs involving a red sock, since there are 13 other colours. The number of unmatched pairs involving a blue sock, *not including the red-blue combination already counted*, is 12. Continuing gives the total number of pairs as $13 + 12 + 11 + \dots + 1 = 91$. Gauss's trick of writing the sum backwards, or equivalently pairing all terms except the middle one, can be used to arrive at $(13 \times 14)/2$ as the sum, hence avoiding actually adding up the 13 numbers. "n choose 2" can be hinted at here, perhaps.
13. Answer: \$30 million. After giving his best friend $1/11$ of the jackpot, Mr. Brown has $10/11$ remaining. After giving the second-best friend $1/12$, he has $11/12$ of $10/11$ remaining, i.e. $(11/12) \times (10/11) = 10/12$ remaining. Continue, multiplying by $12/13$, $13/14$, ..., $19/20$. Noting the cancellation, we see that Mr. Brown retains $10/20$ or $1/2$ of the jackpot.
14. Answer: 4. First note that the number of loonies must be divisible by 24, since 24 is the least common multiple of 3 and 8, and similarly the number of toonies must be divisible by 12. Moreover, since $(1/3) \times 24 + (1/12) \times 12$ equals 9 and so does $(1/8) \times 24 + (1/2) \times 12$, a jackpot of 24 loonies and 12 toonies fits the bill, and that gives $(24 + 12)/9 = 4$ grandchildren. In fact the jackpot can be any multiple of this configuration (e.g. 48,00,000 loonies and 24,000,000 toonies to make our meagre Lotto 21 more lucrative than problem 13's Lotto 649), but no matter the multiple there will still be 4 grandchildren.
15. Answer: 11. It'll take at least 6 games before any player runs out of money, so for each of the first 6 games the pot grows by \$3. At the end of the 6th game, there is just $4 \times \$6 - \$18 = \$6$ still in the hands of the 4 players. The maximum number of games will result from having the pot grow most slowly thereafter, and with at least two players staying alive as long as possible. If the \$6 is distributed among two players each having \$3 (so that the other two players lost all the games and each of these two players won 3 games), then the pot grows by just \$1. And if the two players take turns winning, so they both last as long as possible, then 5 additional games are needed.
16. Answer: 6 m^2 . Pretend the overlapping portion of one of the carpets is cut out, and half of it is further cut up into small pieces and distributed over the uncovered floor. Then the room, which has area $3 \times 9 + 4 \times 4 = 43 \text{ m}^2$, is entirely carpeted (but pretty badly!), with an extra piece leftover that has half the area of the overlap. This extra piece has area $22 + 24 - 43 = 3 \text{ m}^2$, so the overlap has area 6 m^2 .