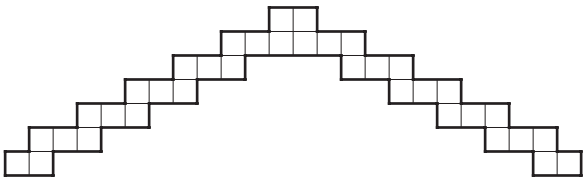
UBC Workshop Problems B

1. At a banquet, there was one serving dish of rice for every three people, one serving dish of vegetables for every four people, and 2 serving dishes, one of duck and one of fish, for every six people. Altogether, 88 serving dishes of food were set out. How many people were at the banquet?

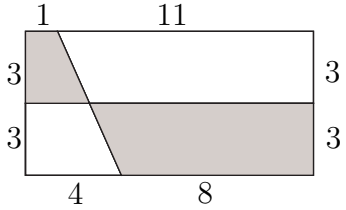
2. The figure below was put together from thirty-six little squares. The area of the figure is 100 square centimetres. What is the perimeter of the figure?



3. The vertices of square, taken counterclockwise, are $A(1, 7)$, $B(s, t)$, and $C(15, 3)$, and D. Find (s, t).

4. Show how to cut up a square into (i) 9 squares; (ii) 10 squares; (iii) 11 squares; (iv) 2005 squares.

5. Find the shaded area.



6. In how many different ways can 11 identical muffins be distributed among A, B, and C if each of them must receive at least one muffin?

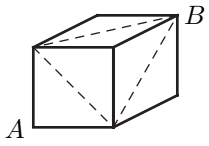
7. Alva left all her money to her three grandchildren, Beti, Cecil, and Delbert. Beti got half the money, plus \$1000. Cecil got half of what was left after that, plus \$2000. And Delbert got the remaining \$5000. What was the total amount of money Alva left?

8. How many ways are there to label the six faces of a cube with the labels 1, 2, 3, 4, 5, 6 ? Two labellings are different if one can't be obtained from the other by a rotation of the cube. (Real dice have their faces labelled so that the numbers on opposite faces always add up to 7, but we are not making that restriction here.)

9. There are altogether 15 positive integers that divide 400. Find the product of these 15 numbers.

10. Candles A and B each burn at a uniform rate. But because A is thicker than B, it burns down more slowly. The candles were lit at 7:00. At 8:00, the candles were of the same height. At 11:00, candle B was finished. And at 12:00, so was A. At 7:00, A was 18 cm high. How high was B?

11. The picture is of a box of the usual shape. Three face diagonals are shown; they have lengths 39, 40, and 41. Find the distance from A to B.



12. Twenty percent of the people who like chocolate like hot pepper. Ninety percent of the people who like hot pepper like chocolate. Everyone likes one or the other or both. What fraction of the people like both?

13. Given that a is a number such that $\left|a - \frac{1}{a}\right| = 1$, what can we conclude about $\left|a + \frac{1}{a}\right|$?

14. Let $\mathcal{A}$ be the set of all points (x, y) such that $|x + y| + |x - y| \leq 4$. Find the area of $\mathcal{A}$.

15. Find all triples (a, b, c) of positive integers such that

$$\frac{1}{a} - \frac{1}{b} + \frac{1}{c} = \frac{1}{2}.$$

16. A restaurant bought 48 kg of swordfish and 48 kg of tuna. If the restaurant had divided the money it spent on fish equally between swordfish and tuna, it would have been able to buy 2 more kg of fish. Given that a kilogram of swordfish costs \$10.00, what can we conclude about the price of a kilogram of tuna?

17. Alphonse ran in a cross-country race, running half of the *distance* at 3 minutes per km and half at 3 minutes 10 seconds per km. If he had run half of the *time* at 3 minutes per km, and half at 3 minutes 10 seconds per km, it would have taken him 1 second less to finish the race. How long did Alphonse actually take?