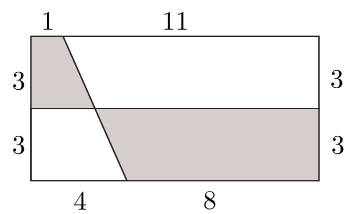
UBC Workshop Problems C

1. What can we conclude about $x + y$ from the equations

$$x^4 - y^4 = 48, \quad x^2 + y^2 = 8, \quad \text{and} \quad x - y = 2?$$

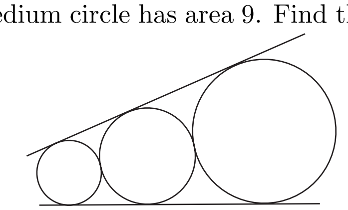
2. Find the shaded area.



3. The vertices of square, taken counterclockwise, are $A(1, 7)$, $B(s, t)$, and $C(15, 3)$, and D. Find (s, t).

4. In how many different ways can 101 identical muffins be distributed among A, B, and C if each must receive at least one muffin? What about if there are four people?

5. The two lines are each tangent to all three circles. The small circle has area 4 and the medium circle has area 9. Find the area of the big circle.



6. Show how to cut up a square into (i) 9 squares; (ii) 10 squares; (iii) 11 squares; (iv) 2005 squares.

7. Sketch the part of the xy-plane where $|y| + |2x - y| \leq 4$.

8. Suppose that $x \geq 0$, $y \geq 0$, and $x^2 + y + 1 = 2xy$. Find the smallest possible value of y. No calculus please!

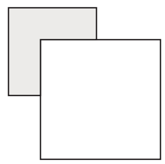
9. Given that a is a number such that $\left|a - \frac{1}{a}\right| = 1$, what can we conclude about $\left|a + \frac{1}{a}\right|$?

10. One bug is condemned to live on the circle $x^2 + y^2 - 6x - 8y = 0$, another on $x^2 + y^2 + 16x - 12y + 64 = 0$. How close can they can get to each other?

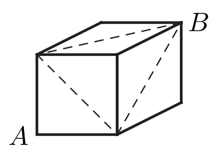
11. Find all quadruples (a, b, c, d) of integers with $0 < a < b < c < d$ and

$$\frac{1}{a} + \frac{1}{b} + \frac{1}{c} + \frac{1}{d} = 1.$$

12. Glen wants to add a symmetrical L-shaped deck to his square cottage. He got a good deal on 18 metres of decorative fencing for the outside boundary of the deck (there is no fencing along the walls of the cottage). What is the largest possible area of the deck?



13. The picture is of a box of the usual shape. Three face diagonals are shown; they have lengths 39, 40, and 41. Find the distance from A to B.



14. Alphonse ran in a cross-country race, running half of the *distance* at 3 minutes per km and half at 3 minutes 10 seconds per km. If he had run half of the *time* at 3 minutes per km, and half at 3 minutes 10 seconds per km, it would have taken him 1 second less to finish the race. How long did Alphonse actually take?

15. How many sequences a_0, a_1, a_2, a_3, a_4, a_5 of six non-negative integers are there such that (i) any number in the sequence is the sum of the previous two, and (ii) $a_5 = 2005$?

16. The volume of the top 27 meters of an Egyptian-style pyramid is equal to the volume of the bottom 1 meter. Find the height of the pyramid.

17. Let $\mathcal{S}$ be an infinite strip of paper of width 1. What is the largest number A such that any triangle of area A can be completely covered by $\mathcal{S}$?