

UBC Workshop Problems B

1. Three runners compete in a 100 meter race. How many possible orders of finish are there, if ties are allowed?
2. An arena has 21000 seats. It is divided into four sections. Section A has twice as many seats as Section B. Section C has twice as many seats as Section D. Section B has 1000 more seats than Section D. How many seats are in each section?
3. A sports league has two conferences, East and West. Each conference has 10 teams. Every year, each team plays every team in its conference twice and plays every team in the other conference once. What is the total number of games played in the league during the year?
4. A, B, C, and D are running a marathon along a straight road. As usual, A is in front, B is next, C is behind B, and D is behind C. At this instant, A is 1 mile ahead of C, B is 4 times as far from A as she is from C, and D is also 4 times as far from A as she is from C. What is the distance, in miles, between B and D?
5. A poster is 40 centimeters wide. There are two pictures on the poster. Each picture is 25 cm wide and 20 cm high. Together the pictures take up one-third of the area of the poster. How many centimeters are in the height of the poster?
6. How many positive integers are factors of 720? Here are a few of them: 1, 5, 8, 360, 720.
7. A gambler is allowed to toss a fair coin six times. She wins if during the tossing she gets three or more heads in a row or three or more tails in a row. What is the probability that the gambler wins?
8. Tom has a total of \$3.30 in nickels and dimes. If his nickels were dimes and his dimes were nickels, he would have \$4.80. How many nickels and how many dimes does Tom have?
9. Let $x * y = \frac{x}{x + y}$. If $x * y = 9$, what is $y * x$?
10. The plane was full when it left Vancouver. In Seattle, half the people got off and 28 got on. In Portland, half the people got off, 40 got on, and the plane was full again. How many people were on the plane when it left Vancouver?
11. If n is a positive integer, then $n!$ (read this as “ n factorial,” or “factorial n ”) is the product of all the numbers from n down to 1. For example,

$$4! = 4 \times 3 \times 2 \times 1 = 24, \quad 5! = 5 \times 4 \times 3 \times 2 \times 1 = 120.$$

Find the highest power of 2 that divides $32!$. For example, the highest power of 2 that divides $5!$ is 2^3 .

12. A paper drinking cup is cone-shaped. When there is water in the cup to a depth of 4 inches, the cup contains 16 cubic inches of water. How many cubic inches of water are in the cup when the water is 3 inches deep?
13. Ten consecutive odd integers add up to 800. What is the smallest of these integers? An example of 10 consecutive odd integers is 7, 9, 11, 13, 15, 17, 19, 21, 23, 25—but they don't add up to 800.
14. How many ways are there to write down three numbers a , b , and c chosen from the numbers 1, 2, 3, 4, $\dots$, 8, 9 so that $a < b < c$ and $a + b + c$ is a multiple of 3? (One such triple of numbers is 4, 6, 8.)
15. The interior of cooking pot A is a cylinder with base diameter 15 cm and height 10 cm. The interior of cooking pot B is a cylinder with base diameter 30 cm and height 40 cm. Pot A is filled with water and the contents are poured into pot B. After this has been done a total of six times, how many cm deep is the water in pot B?
16. Three swimmers had a race across a small lake. Each swam at constant speed. When A finished, she was 20 metres ahead of B, and 40 metres ahead of C. When B finished, she was 20.5 metres ahead of C. Over how many metres was the race?
17. Al and Bob are having a two lap race in a 30 metre pool. Al swims the first lap freestyle at 2 metres per second. For the second lap he swims the backstroke at 1 m/s. Bob swims the butterfly at 1.5 m/s for the entire race. At what time(s) after the start will Al and Bob be side by side? (Al and Bob are very small—in fact they are points.)