

UBC Workshop Problems A

1. Alicia writes down her name over and over like this:

AliciaAliciaAliciaAliciaAliciaAliciaAliciaAliciaAlicAlic...

What is the 99th letter that she writes down? What is the 999th?

2. Erin, Lynn, and Tina bake one apple pie and one cherry pie. Erin eats one-half of the apple pie and one-quarter of the cherry pie. Next, Lynn takes one-half of what is left of the apple pie and one-third of what is left of the cherry pie. How much of each pie is left over for Tina?
3. A 4 cm by 4 cm by 4 cm wooden cube is painted red all over. It is then cut up into $1 \times 1 \times 1$ cubes. How many of the little cubes are there? How many have one red face? Two red faces? Three red faces? No red faces?
4. What fraction is halfway between $\frac{1}{4}$ and $\frac{1}{3}$?
5. A sports league has two conferences, East and West. Each conference has 10 teams. Every year, each team plays every team in its conference twice and plays every team in the other conference once. What is the total number of games played in the league during the year?
6. An arena has 21000 seats. It is divided into four sections. Section A has twice as many seats as Section B. Section C has twice as many seats as Section D. Section B has 1000 more seats than Section D. How many seats are in each section?
7. Lisa's bag of groceries costs \$19.53. She pays with a \$20 bill. The cashier has many quarters, nickels, and dimes in his till, but only 9 pennies. How many different combinations of coins can Lisa get as change?
8. Three runners compete in a 100 meter race. How many possible orders of finish are there, if ties are allowed?
9. A poster is 40 centimeters wide. There are two pictures on the poster. Each picture is 25 cm wide and 20 cm high. Together the pictures take up one-third of the area of the poster. How many centimeters are in the height of the poster?
10. Amy, Mark, and Suzy together can just manage to pull a 140 kilogram sled. Mark can pull twice as much weight as Suzy, and Amy can pull twice as much weight as Mark. Up to how much weight can Suzy pull all by herself?
11. If Ken gives Joey one of his quarters, they will have the same number of quarters. If Joey gives Ken one of her quarters, Ken will have twice as many quarters as Joey has. How many quarters do they each have?

12. How many positive integers are factors of 720? Here are a few of them: 1, 5, 8, 360, 720.
13. The plane was full when it left Vancouver. In Seattle, half the people got off and 28 got on. In Portland, half the people got off, 40 got on, and the plane was full again. How many people were on the plane when it left Vancouver?
14. Ten consecutive odd integers add up to 800. What is the smallest of these integers? An example of 10 consecutive odd integers is 7, 9, 11, 13, 15, 17, 19, 21, 23, 25—but they don't add up to 800.
15. A, B, C, and D are running a marathon along a straight road. As usual, A is in front, B is next, C is behind B, and D is behind C. At this instant, A is 1 mile ahead of C, B is 4 times as far from A as she is from C, and D is also 4 times as far from A as she is from C. What is the distance, in miles, between B and D?
16. The interior of cooking pot A is a cylinder with base diameter 15 cm and height 10 cm. The interior of cooking pot B is a cylinder with base diameter 30 cm and height 40 cm. Pot A is filled with water and the contents are poured into pot B. After this has been done a total of six times, how many cm deep is the water in pot B?
17. Three swimmers had a race across a small lake. Each swam at constant speed. When A finished, she was 20 metres ahead of B, and 40 metres ahead of C. When B finished, she was 20.5 metres ahead of C. Over how many metres was the race?
18. Suzie has \$1.55—1 loonie, 1 quarter, 2 dimes, and 2 nickels. She sees a dime and a nickel on the ground and picks one of them up. While picking up her new coin, she drops one of her old coins. What is the probability that she still has \$1.55?
19. A 20 cm by 20 cm square cake is 7 cm high. It has smooth icing on top and apricot glaze on the sides. Divide the cake among 5 people so that cake, icing, and glaze are all divided equally. Don't use a potato masher. Hint: The midpoint of the top of the cake is involved.
20. If n is a positive integer, then $n!$ (read this as “ n factorial,” or “factorial n ”) is the product of all the numbers from n down to 1. For example,

$$4! = 4 \times 3 \times 2 \times 1 = 24, \quad 5! = 5 \times 4 \times 3 \times 2 \times 1 = 120.$$

Find the highest power of 2 that divides $32!$. Note that for example the highest power of 2 that divides $5!$ is 2^3 .