

UBC Workshop Problems C

1. Let $P = (5, 0)$, $Q = (4, 4)$, and $R = (0, 5)$. Find the area of $\triangle PQR$.
2. A 10 metre by 16 metre pool is surrounded by a walkway of uniform width whose area is 87 square metres. How wide is the walkway?
3. A sports league has two conferences, East and West. Each conference has 10 teams. Every year, each team plays every team in its conference twice and plays every team in the other conference once. What is the total number of games played in the league during the year?
4. A paper drinking cup is cone-shaped. When there is water in the cup to a depth of 4 inches, the cup contains 16 cubic inches of water. How many cubic inches of water are in the cup when the water is 3 inches deep?
5. Find all (real) values of k such that $x^2 - 2kx + k + 1 = 0$ has no real roots.
6. A 6×4 (base 6, height 4) rectangle is divided into twenty-four 1×1 squares by drawing 3 lines parallel to the base of the rectangle and 5 lines perpendicular to its base. How many different rectangles can be formed using one or more of the 1×1 squares?
7. Solve for x : $(x^2 - 6x)(x^2 - 6x + 6) = 16$.
8. Al and Bob are having a two lap race in a 30 metre pool. Al swims the first lap freestyle at 2 metres per second. For the second lap he swims the backstroke at 1 m/s. Bob swims the butterfly at 1.5 m/s for the entire race. At what time(s) after the start will Al and Bob be side by side? (Al and Bob are very small—in fact they are points.)
9. The bisector of one of the acute angles of a right-angled triangle divides the opposite side into segments of length 7 and 25. Find the area of the triangle.
10. Find all pairs (x, y) of positive integers such that $xy = 4x + 5y + 6$.
11. A cat owns 4 identical socks and 4 identical boots. In how many orders can it put on socks and boots in the morning? There is no such thing as a left cat boot or a right cat boot. And a sock must go on a paw before—but not necessarily immediately before—a boot is put on that paw.

- 12.** (i) Find integers a and b such that $0 < a - b\sqrt{2} < 0.6$. (ii) Find integers a and b such that $0 < a - b\sqrt{2} < 0.36$. (iii) Find integers a and b such that $0 < a - b\sqrt{2} < 0.01$.
- 13.** Let P be the point with coordinates $(4, 6)$ and Q the point with coordinates $(0, 3)$. Find the coordinates of the point(s) R on the x -axis such that $\triangle PQR$ has area 20.