UBC Grade 6–7 Workshop Solutions 2003

1. Encourage students to draw a picture, and ensure that they understand the difference between 1/3 of what is left, and 1/3 of the entire pizza. Divide the remaining 1/2 pizza into three equal pieces. After explaining the solution in pictures, it can be useful to repeat the solution using fractions, while referring to the picture: $1 - \frac{1}{2} - \frac{1}{3}\left(\frac{1}{2}\right) = \cdots = \frac{1}{3}$.

2. We can find out how many words each person types in 12 minutes, and subtract to find the difference. If Mr. Wang can type 40 words in 1 minute, how many words can he type in 2 minutes? in 3 minutes? The total number of words can be found by multiplying. Mr. Wang would type $40 \times 12 = 480$ words, Mr. Wong would type $25 \times 12 = 300$ words, so Mr. Wang would be ahead by $480 - 300 = 180$ words. It can then be pointed out that with somewhat less work, the answer can also be calculated as $(40 - 25) \times 12 = 180$.

3. Find out how many good apples are left, and divide that number by the number of people in the family. One-quarter of 60 apples is $\frac{1}{4}(60) = \frac{60}{4} = 15$ apples, so there are $60 - 15 = 45$ good apples. There are 5 people in the family. Is 45 divisible by 5? Each person gets $45/5 = 9$ apples.

4. It may first be necessary to explain how a balance works. A picture helps. From the given information, we can find how much three cans of pop weigh, and divide this number by three to find the weight of one can. Three cans of pop weigh $\frac{5}{8}$ kg, so one can weighs $\frac{5}{8} \times \frac{1}{3} = \frac{5}{24}$ kg. Operations with fractions may need to be reviewed, in particular dividing by 3 is the same as multiplying by $\frac{1}{3}$.

5. Find the proportion of Blue Eyes White Dragon cards there are in the remaining cards. There are $10 - 2 = 8$ cards remaining, including $5 - 1 = 4$ Blue Eyes White Dragon cards. So Edmond has 4 chances in 8, or a 50% probability, of picking a Blue Eyes White Dragon card from the remaining cards.

6. The definitions of a square, and its perimeter and area may need to be reviewed. The area of Square A can be found from its perimeter, then the area of Square B can be found, then its perimeter. Since a square has four sides of equal length, a side of Square A has length $12/4 = 3$ cm. Then the area of Square A is $3 \times 3 = 9$ cm^2 (a diagram showing this should be drawn). The area of Square B is four times the area of Square A, $4 \times 9 = 36$ cm^2. What are the dimensions of Square B? What number times itself equals 36? A side of Square B has length 6 cm, so the perimeter is $4 \times 6 = 24$ cm. A nice way to solve this, which could be pointed out afterwards, is to draw Square A in one corner of Square B, occupying one-quarter of the area of Square B.

7. Out of four equal parts of mixed juice, three parts are water and one part is concentrate. Find out how much 4/5 of the jug can be divided into four equal parts, 1/5 of the jug each. A picture of the jug divided up this way should be drawn. How much is 1/5 of 2000 mL? How much is 3/5 of 2000 mL? You need $3 \times 400 = 1200$ mL of water. Check that $1200 + 400 = 1600$ mL of mixed juice does indeed fill 4/5 of the 2000 mL jug. The problem could be solved algebraically, but a visual solution is more convincing and should be given priority.

8. Find out how much each way will cost, and compare. In the first case the regular price is \$400 and 30% is taken off. That is, \$30 is taken off for every \$100 of the regular price. So, 4×\$30=\$120 is taken off the regular price of \$400, and Frank would have to pay \$400−\$120=\$280. In the second case, 25% is taken off the regular price of \$375, so the amount taken off is $\frac{25}{100}$×\$375=\$93.75. Therefore Frank would have to pay \$375.00−\$93.75=\$281.25, and he would get a better deal in the first case, getting the video game through his mother. Alternatively, one could compare 0.7×\$400=\$280 with 0.75×\$375=\$281.25.

9. A table could be made, showing the population in each week. After 1 week there are $300 \times 2 = 600$ ants, after 2 weeks there are $600 \times 2 = 1200$ ants, after 3 weeks there are $1200 \times 2 = 2400$ ants, etc. After filling in some of the table, it could be pointed out that $600 = 300 \times 2$, $1200 = 300 \times 2 \times 2 = 300 \times 2^2$, $2400 = 300 \times 2 \times 2 \times 2 = 300 \times 2^3$, and so forth. So after 8 weeks there are $300 \times 2^8 = 300 \times 256 = 76,800$ ants. To do the problem the first and most straightforward way would be somewhat lengthy and tedious, so recognizing patterns can save work.

10. Find out how much water each person must add to their jug, and compare. Draw a picture to illustrate the jugs (rectangles are best for making visual connections with the calculations),+ their current contents and the levels needed to reach three-quarters full. Keenan's 60 L jug has $\frac{1}{3}(60) = 20$ L to start, and needs to be filled to $\frac{3}{4}(60) = 45$ L, so $45 - 20 = 25$ L must be added. Lucy's 48 L jug starts with $\frac{1}{4}(48) = 12$ L and must be filled to $\frac{3}{4}(48) = 36$ L, so $36 - 12 = 24$ L must be added. Therefore Keenan needs 1 L more water.

11. The information needs to be organized carefully to answer the question. A diagram, such as a tree diagram, is recommended. Of the 35 students, 18 handed in their assignment, so $35 - 18 = 17$ did not. Of these 17 that did not hand in their homework assignment, 12 had no math exam, so $17 - 12 = 5$ students had a math exam and did not hand in homework. Since 20 students in total had a math exam, $20 - 5 = 15$ had a math exam and did hand in their homework.

12. There are 171 goals total and we know how many Mike scored; Ned scored two-thirds of the remaining goals. Subtracting Mike's goals from the total gives $171 - 60 = 111$. If Ned scores twice as many goals as Owen, how can we divide up the 111 remaining goals to find how many Ned and Owen each scored? Is 111 divisible by 3? $111/3 = 37$ so Owen scored 37 goals and Ned scored twice as many: $2 \times 37 = 74$ goals. Check that $60 + 37 + 74 = 171$. A pictorial representation helps. An algebraic solution (e.g., let x be the number of goals Owen scores, etc.) could be done also, if appropriate.

13. To answer the question, we find out how many packages Elissa sells, and how much profit she makes on each package. For each package, the profit is 20% of 185 cents (20 cents profit for every 100 cents paid to the wholesaler) or $20 \times \frac{185}{100} = 37$ cents, and therefore Elissa sells the packages for $185 + 37 = 222$ cents, or \$2.22. How many packages sold at \$2.22 give \$1887 worth of sales? Dividing (a calculator would be useful here) 1887/2.22 gives 850 packages sold. Finally, to find how much profit she makes we multiply $850 \times 37 = 31450$ cents, or \$314.50. This problem requires several steps to solve, and arithmetic operations involving multiple-digit numbers.

14. Since Frank runs at the same rate as Edward but slower than Gillian, and bikes at the same rate as Gillian but slower than Edward, he will not win. We need to find how

long it takes Edward and Gillian to each complete the race, and compare. How long does it take Edward to run 36 km, running at 9 km per hour? A diagram showing the 36 km run, and the 9 km covered in one hour can be used to motivate the calculation: Edward takes $36/9 = 4$ hours. Similarly, he takes $120/30 = 4$ hours to bike the 120 km, and his total time is 8 hours. Gillian takes $36/12 = 3$ hours to run 36 km, and $120/24 = 5$ hours to bike 120 km, and her total time is also 8 hours. The race ends in a tie between Edward and Gillian! Students may find it difficult to manipulate information about rates unless they are given simple examples to motivate the method of calculation.

15. Algebra seems to give the most efficient solution, although at this level "guess and check" is a reasonable and commonly used strategy. If x is Jon's age now, then Laura's age is $4x$ and Jon's age in 4 years is $x + 4$. Laura's age in 4 years is $4x + 4$, and we are told that $4x + 4 = 2(x + 4)$. Solving for x (which should be done carefully showing all steps) gives $x = 2$ so Jon is now 2 and Laura is 8. Check that in 4 years, Jon will be 6 and Laura will be 12, which is twice Jon's age.

16. A tree diagram will help keep track of the different outcomes. At first we can have H or T. If H, then the choices are W_1 or R_1, and if R_1 is chosen then the next choice must be R_2, and the following choice is forced to be W_1 and the game ends. If T, then candy chosen must be R_2, and the next choice is either W_1 or R_1. If R_1 is chosen the next choices must be R_2 and then W_1. So the four different outcomes can be labelled HW, $HRRW$, TRW, $TRRRW$. Keeping track of which bag the candies came from does not affect the number of outcomes.

17. The number of different combinations can be counted, if the counting is done systematically. A tree diagram or similar visual representation should be used. Since the red and green bulbs are always adjacent, treat them as one object initially. Then we have four objects to be placed in four slots. In the first slot there are four possible choices. After making the first choice, then in the second slot there are three possible choices. After making the first and second choices for the third slot there are two possible choices, and in the last slot only one choice. All the choices can be counted, and there are $4 \times 3 \times 2 \times 1 = 24$ possibilities treating the red and green bulbs as one object. But the red and green bulbs can be in either order so in all there are $24 \times 2 = 48$ possibilities.

18. We need to keep track of how many new, day old, and deceased glibbles there are on each day, and discover a pattern. A chart should be made showing the day, the number of new glibbles, the number of day old glibbles and the number of deceased glibbles. On Day 1 there is one new glibble. On Day 2 the original glibble has grown one day old and has given birth, so there is one new glibble and one day old glibble. On Day 3 all the glibbles from the previous day have given birth and grown one day older, so there are two new glibbles, one day old glibble and one deceased glibble. On each day the number of new glibbles is the sum of the previous day's numbers of new glibbles and day old glibbles, while the number of day old glibbles is the previous day's number of new glibbles. We don't really need to keep track of deceased glibbles. Continuing in this way the chart can be filled out to find on Day 9 there are 34 new and 21 day old glibbles, for a total of 55. The number of new glibbles on each day follows the Fibonacci sequence, and so does the number of day old glibbles, shifted by one day.