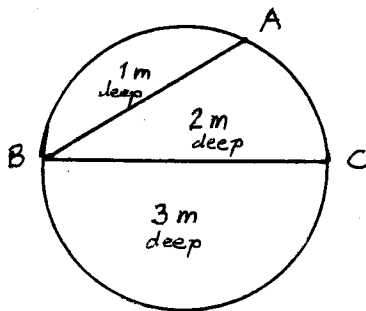


UBC Math 11/12 Workshop Problems 2003

1. A grocer mixes two grades of coffee worth \$14.00 and \$16.00 per kilogram, respectively. How much of each grade must she use to make a 50 kilogram mixture worth \$15.20 per kilogram?
2. Ed and Fred race their bicycles against each other, but Ed travels slower than Fred. Ed rides his mountain bike at 25 km/h and Fred rides his road bike at 32 km/h. The race is 50 km, and Ed gets a half hour head start. Who wins?
3. Mark has a string of Christmas lights with five empty sockets in a row, and a package of five new bulbs coloured red, green, blue, orange and yellow. Because it is Christmas, Mark wants the red bulb to always be next to the green bulb. If he follows this rule, how many different ways can Mark put all the new bulbs in all the empty sockets?
4. Professor Warner has discovered a new animal called a glibble. Each glibble lives for only two days. One day after it is born, it gives birth to a new glibble, and two days after it is born it gives birth to one more new glibble and dies. If the professor starts with one newborn glibble at the beginning of the first day, how many live gibbles would he have in the middle of the fifteenth day?
5. A car is travelling at 30 m/s along a road, and a limousine is travelling at 24 m/s in the opposite direction. If the car is 3 m long and it takes $\frac{1}{6}$ s for the car and the limousine to completely pass by each other, how long is the limousine?
6. A large building is in the shape of a cylinder 100 m high. The circular roof has a diameter of 60 m. A staircase spirals once around the outside of the building with the top of the staircase exactly above the bottom of the staircase. How long is the staircase?
7. A spherical alien spaceship crashed into the ocean and partially buried itself in the flat sand on the ocean floor. Divers found a hump of the sphere sticking $\frac{1}{2}$ m out of the ocean floor with a diameter of 8 m. What is the volume of the entire spaceship?
8. As seen from above a circular swimming pool has its deep (3 m), medium (2 m) and shallow (1 m) sections divided as shown in the diagram. The diameter BC is 36 m,



and the angle ABC is 30° . How much water does it take to fill the pool?

9. A drinking cup in the form of a right circular cone is made from a circular disk of paper of diameter 20 cm. A sector of angle 72° is cut out of the disk and the remaining part

of the disk is bent up so that the two straight edges join without overlapping and a cone is formed. How much water can the cup hold?

10. What is the last digit of 3^{2003} ? Prove your answer.
11. How many in the set of numbers $\left\{ \frac{1}{10000}, \frac{2}{10000}, \frac{3}{10000}, \dots, \frac{9999}{10000}, \frac{10000}{10000} \right\}$ are fractions reduced to lowest terms?
12. Katie has 80 m of fencing to make a rectangular enclosure. One side of the enclosure will be all or part of one side of Katie's house, which is 20 m long, and one corner of the enclosure will be at a corner of her house. What is the largest rectangular area that she can enclose?
13. A store is having a sale on pants, shirts and sweaters. The regular prices of all the pairs of pants on sale are the same, and this is also true of the shirts and of the sweaters. The regular prices of two pairs of pants, two shirts and two sweaters is \$230 in total. For the sale, if you buy two of the same item, the first item is at the regular price and the second is at 20% off, and if you buy three of the same item the third is 30% off. During the sale Irving buys two pairs of pants, one shirt and one sweater for \$155, Jerome buys one pair of pants, three shirts and two sweaters for \$184.50, and Keenan buys three pairs of pants, two shirts and some sweaters for \$270. How many sweaters did Keenan buy? What is the regular price of a pair of pants? a shirt? a sweater?
14. Consider an analogue clock with only an hour hand and a minute hand. Suppose at time T the angle between the two hands of the clock is 45° , and seventy minutes later the value of the minutes showing is five times larger than the value of the hours. What is time T ?
15. Three identical circles, each of area A , are placed in a triangular arrangement so that they are as close together as possible with their edges just touching. What is the area enclosed in between the circles? (Express your answer in terms of A .)
16. Two friends agree to meet for dinner at the White Tops restaurant, but they don't arrange an exact time. They only agree to arrive some time between 6:00 p.m. and 7:00 p.m. If the time that they each arrive is random (*i.e.* all times between 6:00 and 7:00 p.m. are equally likely), what is the probability that the two friends arrive within 15 minutes of each other?
17. The product of three consecutive prime numbers is 1113121. Find the product of the smallest and the largest of the three prime numbers.
18. Find the largest integer power of 2 that divides $100!$.