# UBC Grade 6–8 Workshop Problems, 2002

**1.** If 1 horse weighs as much as 15 dogs, and 3 dogs weigh as much as 8 cats, then 5 horses weigh as much as how many cats?

**2.** Billy is 7 years old and has $21 to spend. Billy wants to buy a video game on sale for $50 (half the original price) until Sun. Oct. 28. Billy gets paid his allowance of $15 every Saturday but insists on depositing half of that into a long-term account for his future. If today is Friday Oct. 5, will Billy have enough money to buy his video game?

**3.** Your mom is throwing a party for your sister's 10th birthday. She asks you to buy 1 kg of chips for the occasion, hands you a ten-dollar bill, and says that you may keep the change. Being as clever as you are, you glance through 3 different flyers to scope out the best deal. The advertisements are as follows: Walmark—47 cents for a 50 gram bag; Safepay—two 200 g bags only $3.33 ($2.25 for one); Costcor—1000 g bag $9.28. Which store should you go to?

**4.** Carla buys 2 pizzas, one vegi and one pep. Carla gives 1/6 of the vegi pizza to Meg and Meg gives 3/4 of her vegi pizza to Jama. Carla gives 3/8 of her pep pizza to Rochelle and Rochelle gives 1/3 of her pep pizza to Jama. How much pizza in total does Jama have?

**5.** Mary lives in an apartment with 2 roommates, Jenny and Susan. Their rent each month is $900. Mary pays 1/3 of the rent. Jenny pays 50% of what Susan pays. How much does each pay per month?

**6.** Mike likes basketball. He can score three points from the 3-point line, two points by lay-ups, and one point from the free throw line. Mike practices a lot… He takes 100 shots from each spot and makes 30% from the 3-point line, 70% of his lay-ups, and 80% of his free throws. He gets to go to McDonalds if he makes more than 50% of the total points he would have made if he had made every shot. Does Mike get to go to McDonalds?

**7.** Mr. Ina Rush is in a rush on his way to work. He grabs two socks from his socks drawer at random, so he can put them on while sitting on the bus. Originally (before Mr. Rush grabs any socks) there were ten loose socks in the drawer: 6 black socks and 4 green socks. Five of these ten socks had holes in the toe, and only one green sock did not have any holes. As it turns out, the first sock Mr. Rush picks is a black sock with no holes. Given this, what are the chances Mr. Rush has a matching pair of socks without any holes? (By the way, Mr. Rush forgot his shoes!)

**8.** There is a certain number of students in a class. To begin with, the teacher places them in complete rows of seven. Whenever they have to pair up, someone always has to pair with the teacher (ew!). Later on that year a new student joins the class. The teacher can no longer have rows of seven, but instead rearranges them into groups of exactly three students. What was the smallest number of students in the original class?

**9.** Brian has a defective 12-hour clock: every 6 minutes, the clock loses one minute. If Brian's clock reads the correct time right now, when will it read the correct time again?

**10.** You are given a piece of paper that is 24 cm wide and 40 cm long. You fold the paper in half widthwise and then fold it in half again 3 times, folding it in half widthwise a total of 4 times. You then fold it in half lengthwise 3 times. How many times larger is the area of the paper before it is folded than after it is folded?

**11.** There are 15,000 fans at a baseball game. Tickets for the game were $24, $25, or $30. If 2/3 of the fans paid $25 and 1/5 of the fans paid $30, how much money in total was spent on tickets?

**12.** When filled, a ferry can carry 500 cars and 400 bicycles, or 600 cars, or 400 bicycles and 3000 passengers. If the ferry only carries passengers, what is the maximum number of passengers it can carry?

**13.** Each row of hearts has 2 more than the row before. Find the total number of hearts in the first 20 rows.

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**14.** You're holding a party for your friends and are not sure if 5 or 6 people will show up. You have one super large pizza and want to precut it in a way that either 5 or 6 people can take equal amounts. One way is to cut the pizza into 30 equal slices. Is there a way of cutting fewer slices (which may not be equal)?

**15.** Is the product of 1,000,000,001 and 999,999,999 less than, greater than, or equal to 1,000,000,000,000,000,000?

**16.** Alan enters a judo tournament with the rule that if a contestant loses 2 matches he is thrown out. There are 41 matches, 4 of which are draws, before Alan wins the competition. How many judo players entered the tournament?

**17.** If 500…00 (5 followed by 50 zeros) is divided by 101, what is the first digit to the right of the decimal point?

**18.** Neopolitan Bonaparte has three flavours of ice cream in his freezer: vanilla, chocolate, and strawberry. He wants to make himself an ice cream cone with 5 scoops of ice cream (piled one on top of the other). How many different ways can Neopolitan arrange the scoops of ice cream so that (1) no two scoops next to each other on the pile are of the same flavour and (2) there is at least one scoop of every flavour?

# UBC Grade 6–8 Workshop Solutions, 2002

**1.** 15 dogs weigh as much as $5 \times 8 = 40$ cats, so 1 horse weighs as much as 40 cats and 5 horses weigh as much as $5 \times 40 = 200$ cats. Note that the problem can be done *without* using fractions.

**2.** Billy will get 4 allowances before Oct. 28—on Oct. 6, 13, 20, and 27. From each allowance, he will be able to spend half, or \$7.50. So, Billy will have $\$21 + 4 \times \$7.50 = \$51$ to spend on Oct. 27, and he will be able to buy the video game (assuming there is no tax!).

**3.** Consider all 3 possibilities. At Walmark, you will need to buy twenty 50-gram bags, so you will spend $\$0.47 \times 20 = \$9.40$. At Safepay, you will need to buy four 200-g bags for \$6.66 and then spend another \$2.25 for a 5th bag, so you will spend \$8.91 total. At Costcor, you will spend \$9.28. You are best off going to Safepay.

**4.** Understanding that we need to multiply fractions to figure out how much of each pizza Jama receives, we see that Jama gets $(3/4) \times (1/6) + (1/3) \times (3/8) = (1/8) + (1/8) = 1/4$ of one pizza in all.

**5.** After Mary has paid her \$300, Jenny and Susan together must pay the remaining \$600. Jenny's rent is half of Susan's, so Jenny pays \$200 and Susan \$400. No algebra is needed; students should be able to determine two numbers that add up to 600, one of which is half of the other.

**6.** How many points total are available? Since there are 100 shots in each category, this number is $300 + 200 + 100 = 600$. How many points did Mike make? This number is $30 \times 3 + 70 \times 2 + 80 \times 1 = 310$. Since 50% of 600 is 300, Mike does get to go to MacDonalds! Translating the given information into the points scored is the key here.

**7.** Look at the drawer after the first sock has been picked: it contains 4 green socks and 5 black socks, a total of 9. All the hole-ly socks are still in the drawer, and there are 5–3=2 black socks with holes (since of the 5 hole-ly socks 3 are green), so 3 of these black socks have no holes. The chances of picking one of these 3 socks from the 9 in the drawer are 3/9 or 1/3. Make sure students understand why it is okay to consider the 9-sock drawer after the first sock has been picked.

**8.** The original number of students must be a multiple of 7 and odd. Also, when 1 is added the number must be a multiple of 3. Write down the numbers that satisfy the first condition: 7, 21, 35, 49,… The third one is the first one that also satisfies the second condition, so the smallest possible number of students is 35.

**9.** The clock runs at 5/6 the correct rate. It will read the correct time again when 1/6 times the number of elapsed hours, which is the amount of time it has lost, is a multiple of 12 (since we have a 12-hour clock); this first happens after 72 hours.

**10.** Each fold reduces the relevant dimension by a factor of 2, so the folded paper has width that is $(1/2) \times (1/2) \times (1/2) \times (1/2) = 1/16$ the original width and length that is 1/8 the original. The folded area is $1/16 \times 1/8 = 1/128$ that of the original, so the original area is 128 times the folded area. Encourage students to conduct physical experiments, although the final foldings will be difficult! The key is noting that each fold halves each dimension.

**11.** The number of fans who paid \$25 and \$30 respectively is 2/3 and 1/5 multiplied by 15,000, i.e. 10,000 and 3000. The number of fans who paid \$24 is therefore 2000. The total spent on tickets is $\$24 \times 2000 + \$25 \times 10{,}000 + \$30 \times 3000 = \$388{,}000$. Note that it is easier to determine the *number* of fans who paid \$24 than the *fraction* of such fans.

**12.** We see that 400 bicycles take the same capacity as 100 cars, so 100 cars plus 3000 passengers will fill the ferry. Since 600 cars also fill the ferry, 500 cars "equals" 3000 passengers, so 600 cars equals $3000 \times (6/5) = 3600$ passengers, the capacity of the ferry in passengers. At this level, algebra should be avoided; instead argue in words.

**13.** We immediately spot a pattern: the total number of hearts in the 1st row is 1; there are 4 hearts in the first 2 rows, 9 in the first 3. It is easy to guess that there will be $20^2 = 400$ hearts in the first 20 rows. Encourage the students to convince themselves that this *guess* is actually correct: rearrange the hearts, noting for example that the first 2 rows can be rearranged into a $2 \times 2$ square and the first 3 rows into a $3 \times 3$ square.

**14.** The pizza can be precut into 10 slices: 5 slices of area 1/6 and 5 slices of area 1/30. If 5 people are to share the pizza, each takes a small slice and a large slice; if 6 people are to share then 5 of them take one large slice each and the 6th person takes all of the small slices. Many students should be able to get this problem if they are given the hint: "start with 6 equal slices." For an advanced class, an indication of why 10 is optimal may be in order: If there are 9 or fewer slices and 5 people show up, then someone has to get just one slice, so one slice must be 1/5. But then if 6 people arrive at the same pizza one of them has to get this slice, but then that person has more than 1/6 of the pizza.

**15.** Note that the numbers are one billion + 1, one billion – 1, and one billion times one billion. Now (one billion + 1)(one billion – 1) equals a billion (billion – 1)'s plus one (billion – 1). On the other hand, (one billion)(one billion) equals (one billion – 1) billions plus one billion, so the first product is one less than the second. Encourage students to work with small examples first. A geometrical argument considering a square of side length N versus a rectangle of sides N+1 and N–1 is also possible. The distributive law, which students will see in later introductions to algebra, is lurking here.

**16.** The key is looking at the total number of losses, which is 37. For each contestant who is thrown out there must have been two losses, and since 37/2 = 18.5 it means there were 18 other contestants, or 19 in total. The extra "0.5" comes from the fact that Alan lost one match.

**17.** Note that 5/101 = 0.049504950495…; for 500…00/101 we shift the decimal point 50 places to the right. Since the decimal expansion has a repeating pattern of 4 digits, we shift through 12 groups of digits and then 2 more, so the first digit after the decimal point will be 9. Encourage students to think about how to take advantage of the special nature of 5 followed by 50 0's. Encourage them to use their calculators, but intelligently!

**18.** There are $3 \times 2 \times 2 \times 2 \times 2 = 48$ arrangements that satisfy (1), since the bottom scoop can be anything but then each subsequent scoop cannot be the same as the one it is put on top of. In these 48 arrangements, if there are at most 2 scoops of each flavour then every flavour is present, so (2) is satisfied. There cannot be 4 scoops of the same flavour since scoops of the same flavour must have another scoop in between, so in the arrangements that satisfy (1) but not (2), scoops 1, 3, and 5 must have the same flavour and scoops 2 and 4 must have a different, common flavour. There are 3 choices for the flavour of scoops 1, 3, and 5 and then two choices for the flavour of scoops 2 and 4. This means $3 \times 2 = 6$ arrangements that satisfy (1) do not satisfy (2), so the number of arrangements for Neopolitan's desert is $48 – 6 = 42$. Students may try to list the possibilities in an ad hoc manner, but it should be stressed that a *systematic* enumeration is necessary in order to ensure that all possibilities are included. Giving a hint: "start by looking at arrangements that satisfy (1) only" may point the students in the right direction.