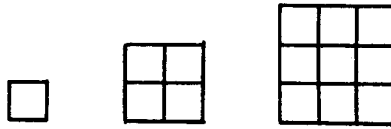


# UBC Workshop Problems 2001

1. How many squares, of various sizes, are in the pictures below? Can you guess how many would be in a  $4 \times 4$  picture? Check your guess.



2. Mega Hit Videos rents videos for \$4.00 each. Movies Plus rents videos for \$3.50 each but also charges \$6.50 for a membership. What is the least number of movies you have to rent for Movies Plus to be a better deal?
3. Marvin's Cellular Phone Service charges 30 cents for the first minute of a call, plus 5 cents for each additional minute. If a call cost \$3.20 how long was the call?
4. Aaron, Bob and Colin order a mushroom and sardine pizza with extra cheese. Aaron takes a quarter of the pizza, and then Bob takes two-thirds of what Aaron left. How much pizza did Aaron and Bob leave for Colin?
5. Fiona has a bag of marshmallows. She gives some to Gavin and keeps 6 for herself. Gavin gives Harold some marshmallows and keeps 3 for himself. Harold gives Inez some marshmallows and keeps 2 for himself. Inez gives Joan 2 marshmallows and keeps the last 4 for herself. How many marshmallows did Fiona originally have?
6. A 900 seat multiplex is divided into three theatres, and there are 370 seats in the first theatre. If the second theatre has 150 more seats than the third, how many seats are in the second theatre?
7. You are in a strange land, and want to go home. You come to a fork in the road. One way leads safely home and the other way leads to danger, but you don't know which way to take. There are two dragons at the fork. One of them always tells the truth, and the other one always lies, but you can't tell which dragon is which. You are allowed to ask only one question, so you cleverly ask one of the dragons, "What would the *other* dragon say if I asked which way leads home?" Then what do you do?
8. Brian can fill his backpack, without leaving any wasted space, with (a) 4 videotapes and 24 CDs, or (b) 3 binders and 4 videotapes, or (c) 5 binders. If he fills his pack with only CDs, how many can he fit in?
9. In order to reach the "hot seat" on the show "Who Wants To Be A Millionaire?" a contestant must answer a "fastest finger question". The question gives you four possible answers A, B, C, D which must be arranged into the correct order. If you just guess at random, what chance do you have to reach the hot seat?
10. A white styrofoam cube has a side of length 3 cm. Three faces of the cube that meet at a corner are painted red, and then the cube is cut into 27 cubes with sides of length 1 cm. How many of these cubes have no painted surface?

11. In a soccer tournament, 10 teams must play every other team exactly once. How many games need to be played?
12. The regular price of a shirt is \$20.00, and it is on sale for 15% off the regular price. If you have to pay a 10% sales tax, what is the final amount you have to pay for the shirt?
13. A fat hose would fill a swimming pool in 4 hours by itself. A medium hose would fill the same pool in 6 hours, and a thin hose would fill it in 12 hours. If all three hoses are used at the same time, how long would it take to fill the pool?
14. What is the 757th number in the sequence 6, 2, 13, 9, 5, 6, 2, 13, 9, 5, 6, 2, 13, 9, 5, 6, 2, 13, 9, 5, ... ?
15. Grapes are 90% water, and raisins are only 40% water. How many kilograms of grapes are needed to make 1 kg of raisins?
16. Suppose you have an even number of quarters. If you put them in groups of five, none are left over, but if you put them in groups of eleven, nine are left over. How many quarters could there be? (There is more than one number possible.)
17. There are five sealed unlabelled cans, each containing either coffee or cocoa or powdered milk.

Can	A	B	C	D	E
Content weight	325 g	475 g	550 g	750 g	950 g

- There is twice as much coffee as cocoa by total weight. No three cans contain the same item, and only one contains cocoa. Which can contains the cocoa? Show why you are sure of your answer.
18. Four squares, all of different areas, are cut from a rectangle, leaving a smaller rectangle of dimensions  $1 \times 2$ . If the largest square has area 64, and the other three squares have side lengths that are whole numbers no larger than 7, what are their areas?

# UBC Workshop Solutions 2001

1. In the first picture there is one  $1 \times 1$  square, in the second picture there is the one  $2 \times 2$  square plus four  $1 \times 1$  squares for a total of  $1 + 4$ , in the third picture one  $3 \times 3$  square, four  $2 \times 2$  squares and nine  $1 \times 1$  squares for a total of  $1 + 4 + 9$ . If the students recognize the pattern rather than (or in addition to) just counting squares, they may be able to guess that a  $4 \times 4$  picture would have  $1 + 4 + 9 + 16$  squares of various sizes. To see if they understand the pattern, ask how many a  $6 \times 6$  picture would contain.
2. With Movies Plus, you save \$0.50 per video, but start out \$6.50 behind. How many videos do you have to rent to save \$6.50? 13 videos. Therefore 14 is the least number of videos to rent for Movies Plus to be a better deal. A table could be used to help organize thinking. (Or focus on the differences in cost after 1, 2, 3 videos, count down by 50 cents until 0 is reached.)
3. After the first minute the call cost  $320 - 30 = 290$  cents, then how many times does 5 cents go into 290 cents? The call was  $1 + (290/5) = 59$  minutes. Trying to use algebra at this level usually doesn't work well. (Some students count by 5's up to 290, or list all the minutes and then count by 5's from 30 to 320)
4. Pictures help. Aaron leaves  $3/4$  of the pizza, Bob takes  $2/3$  of  $3/4$ , or half the pizza, leaving  $1/4$  for Colin. The confusing part for some students is that  $2/3$  of *what Aaron left* is not  $2/3$  of *the pizza*.
5. Work backwards. Some kind of chart or tree could help. Inez received  $2 + 4 = 6$  marshmallows, which means Harold received  $6 + 2 = 8$ . So Gavin received  $8 + 3 = 11$ , and Fiona had  $11 + 6 = 17$  to start. It would be worth running this forwards to check. Some students get the correct answer without knowing why. Encourage them to explain clearly, or at least show that their answer is correct.
6. There are  $900 - 370 = 530$  seats in the second and third theatres. If you took away 150 seats from the second theatre, then it would have the same number of seats as the third theatre, and together they would have  $530 - 150 = 380$  seats. So the third theatre has half this amount,  $380/2 = 190$  seats, and the second theatre has (giving back its 150 seats)  $190 + 150 = 340$  seats. Check by adding  $370 + 340 + 190$ . A diagram can help the explanation.
7. There are only two possibilities, so consider each of them. A chart or diagram could help the explanation. Let H be the way home and D be the way to danger. (a) If you asked the dragon that always tells the truth, then the other dragon always lies and would tell you to take path D. The dragon you asked would faithfully report this and tell you the other dragon would say to take path D. (b) If you asked the dragon that always lies, then the other dragon always tells the truth and would tell you to take path H. The dragon you asked would lie about this and would tell you the other dragon would say to take path D. In either case you should go the other way. Students at this level enjoy acting out the parts, and it helps them understand the logic as well.
8. In terms of space, comparing (a) and (b) we see that  $24 \text{ CDs} = 3 \text{ binders}$ , so  $8 \text{ CDs} = 1 \text{ binder}$ . From (c) we know 5 binders fill Brian's backpack, which takes the same space as  $5 \times 8 = 40 \text{ CDs}$ . A (schematic) picture of each possibility (a), (b) and (c) helps with the explanation.

9. First find all the ways the answers could be arranged. There are 4 possible ways to choose the answer to the first question. For each choice of answer for the first question there are 3 possible ways to choose the answer for the second question. The students need to be convinced that to find how many ways there are to choose answers to the first two questions, you multiply  $4 \times 3$ . This could be done by systematically listing some examples and counting, until the students see the pattern. Continuing, there are 2 possible ways to answer the third question after the first two have been answered, leaving only one way to answer the fourth question. So the total number of different ways to answer the four questions is  $4 \times 3 \times 2 \times 1 = 24$ . Only one of these ways is correct, so choosing at random gives you a 1 in 24 chance to reach the hot seat.
10. Draw a cube showing, for example, the top, front and right side painted. Draw lines to show where the cube is cut. Take away the top (painted) layer of 9 small cubes, the front (painted) layer of 6 small cubes, and the right side (painted layer) of 4 small cubes, leaving 8 small cubes with no painted surface. A sequence of pictures would be helpful. (Or show the three horizontal layers in an exploded view, or count the number of cubes with paint on them and subtract from 27.)
11. Line up the teams. Team 1 must play each of the 9 remaining teams, so this accounts for 9 games. Then team 2 already has a game with team 1, so we need 8 more games, for team 2 to play teams 3 to 10. Team 3 already has games with teams 1 and 2, so we need 7 games with teams 4 to 10. By this time the students hopefully will see a pattern. We need  $9 + 8 + 7 + 6 + 5 + 4 + 3 + 2 + 1 = 45$  games (the sum can be easily done by grouping  $9 + 1$ ,  $8 + 2$ , etc.). A chart could help.
12. The students need to know about percent. Even if they have been taught it, they may need reminding what it means. The sale price is 85% of the regular price,  $(85/100) \times \$20.00 = \$17.00$  (or slightly longer, calculate the discount and subtract from the regular price). The sales tax is 10% of \$17.00, or \$1.70, which is added to the sale price to obtain a total of \$18.70.
13. In one hour, the fat hose would fill  $1/4$  of the pool by itself, the medium hose would fill  $1/6$  of the pool, and the thin hose would fill  $1/12$ . Students may have trouble adding fractions, so it would help to draw a (schematic) pool as a rectangle subdivided into 12 equal-sized regions. The drawing could be used to illustrate that in one hour all three hoses together would fill  $1/4 + 1/6 + 1/12 = 1/2$  of the pool, so would take 2 hours to fill the whole pool.
14. The sequence repeats after every 5 numbers, so the 5th, 10th, 15th, etc. number is 5. The 755th number is 5, so the 756th number is 6 and the 757th is 2. (some students will try to list all numbers to the 757th)
15. Students may need to be told that raisins are (partly) dried grapes. If we could remove all the water from the 1 kg of raisins, we would be left with a pile of crispy totally dry material with a mass of 60% of 1 kg, or  $0.6 = 3/5$  kg. This dry material represents 10% or  $1/10$  of the grapes needed, so we need  $10 \times 0.6 = 6$  kg of grapes to make 1 kg of raisins.
16. The number of quarters is even, so is divisible by 2. The number is also divisible by 5, so must be divisible by  $2 \times 5 = 10$ . We have narrowed down our search of possible numbers of quarters to 10, 20, 30, .... If we subtracted 9 from the number of quarters,

the resulting number would be divisible by 11. But if we subtract 9 from a number divisible by 10, the last digit must be a 1. So what numbers whose last digits are 1 are divisible by 11? Only 11 times some number whose last digit is 1:  $11 \times 1 = 11$ ,  $11 \times 11 = 121$ ,  $11 \times 21 = 231$ ,  $11 \times 31 = 341$ , etc. Adding 9 to each of these gives the possible numbers of quarters: 20, 130, 240, 350, ..., an arithmetic sequence.

17. The correct answer D could be guessed, but students should at least be able to explain why it is correct. Is it the only correct answer? A systematic solution method would be able to show this. Since one can contains cocoa and no three cans contain the same item, there must be two cans containing coffee (and two containing powdered milk). So the problem is reduced to searching for two weights that sum to twice a third, different, weight. All sums of two weights could be found systematically

Sum	A	B	C	D	E
A		800	875	1075	1275
B			1025	1225	1425
C				1300	1500
D					1700
E					

and compared with a list of twice the weights

750, 950, 1100, 1300, 1500, 1900.

The only match is the 1500, so there is only one solution: can D contains the cocoa (and cans C and E contain coffee).

18. The students have to understand that the only possible areas in the answer are among the perfect squares 1, 4, 9, 16, 25, 36, 49. After that, they can experiment with different arrangements. Graph paper would be helpful, and some might enjoy cutting the possible squares out of paper and shifting them around physically.

