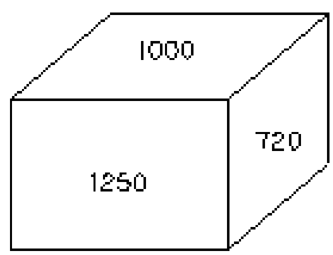
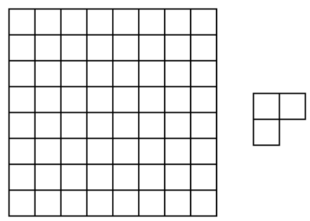
# UBC Grade 11/12 Problems 2000

**1.** A cardboard box of the usual shape has sides of area $720\,\text{cm}^2$, $1000\,\text{cm}^2$, and $1250\,\text{cm}^2$. Find the volume of the box.



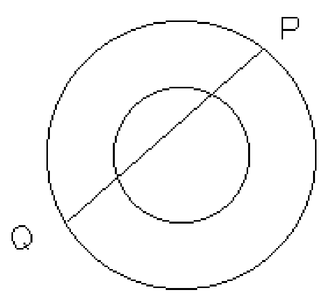

**2.** For what $k$ does the system of equations $x^2 - y^2 = 0$, $(x-k)^2 + y^2 = 1$ have
(i) no solutions;
(ii) one solution;
(iii) two solutions;
(iv) three solutions;
(v) four solutions?

**3.** (a) The chessboard has sixty-four $1 \times 1$ squares. The *tromino* has three $1 \times 1$ squares. In how many ways can we place the tromino on the chessboard so that it covers three squares of the board exactly?
(b) What about if the board has $m$ rows and $n$ columns?



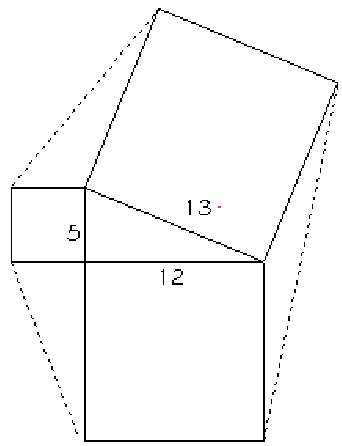
**4.** Alicia started going up the Grouse Grind at 4:30. Fred and Janet started 30 minutes later. Janet passed Alicia at the halfway point, and Fred passed Alicia 16 minutes after Janet did. Janet got to the top 12 minutes before Fred. At what time did Alicia reach the top? (Assume that everyone climbs at unvarying speed.)

**5.** The two circles in the picture have the same center. One of them has radius 1, and the other has radius 2. Points $P$ and $Q$ are chosen at random on the boundary of the outer circle. Find the probability that the line $PQ$ passes through the inner circle.



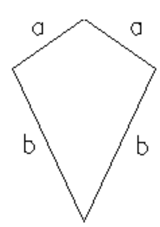

**6.** A rectangular sheet of paper is 10 cm in width and 24 cm in length. It is folded so that two diagonally opposite corners coincide. Find the length of the crease.

**7.** A triangle has sides 5, 12, and 13 (see next page). Outward facing squares are erected on the three sides. Finally, string is placed around the whole figure and drawn tight. Find the area enclosed by the string.




**8.** Let $a > 1$. Write $\cosh_a(t)$ for the function $(a^t + a^{-t})/2$, and $\sinh_a(t)$ for $(a^t - a^{-t})/2$.
(a) If $x = \cosh_a(t)$ and $y = \sinh_a(t)$, find $x^2 - y^2$, simplifying as much as possible.
(b) Express $\sinh_a(2t)$ in terms of $\sinh_a(t)$ and $\cosh_a(t)$.
(c) Let $a = 10$. Solve the equation $\cosh_a(t) = 20$.

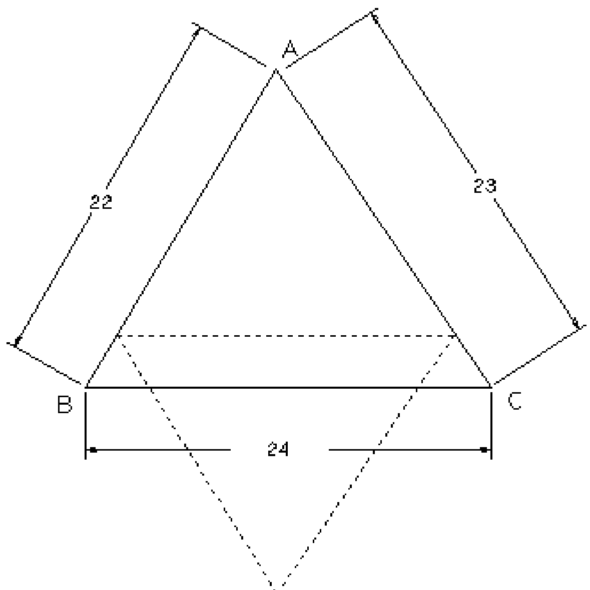
**9.** You have two thin sticks of length $a$ and two of length $b$, and want to make a kite-shaped figure that encloses the largest possible area. What is that maximum area?




**10.** (a) How many ways are there to give change for a ten dollar bill using nothing other than two-dollar coins, one-dollar coins, and quarters?
(b) How many ways are there to give change for a twenty dollar bill? A fifty dollar bill?

**11.** A right-angled triangle has hypotenuse of length 15 cm and has area $16\,\text{cm}^2$. Calculate the perimeter of the triangle.

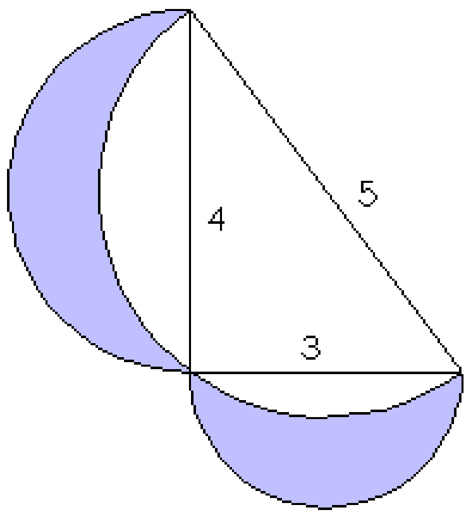
**12.** A triangle $ABC$ with $AB = 22$, $AC = 23$, and $BC = 24$ is cut out of paper. The triangle is folded along a line parallel to $BC$. When this is done, it turns out that a triangle which contains 64% of the area of $\triangle ABC$ sticks out beyond $BC$. Find the length of the fold line.




**13.** The vertices of a regular 12-sided polygon lie on a circle of radius $r$. Let $P$ be one of the vertices. Find the sum of the squares of the distances from $P$ to the other vertices of the polygon.

**14.** The arithmetic progression 9, 32, 55, 78, ... begins with a perfect square. Find the *next three* perfect squares in this progression.

**15.** A right-angled triangle has legs of length 3 and 4. Outward facing semicircles are drawn with these legs as diameter. An inward facing semicircle is drawn with the hypotenuse as diameter. Find the area of the shaded region.




**16.** Find the sum of all the three-digit numbers all of whose digits are odd.

**17.** How far from the origin are the points on the curve $x^4 + 16y^4 = 16$ which are furthest from the origin? (Consider circles $x^2 + y^2 = r^2$ for various values of $r$.)

**18.** Find the first two non-zero digits after the decimal point in the decimal expansion of $(100 + \sqrt{10001})^3$.

**19.** Find an integer $N$ such that the first three digits of $\sqrt{N}$ to the right of the decimal point are 3, 2, and 1 (in that order.)