UBC Grade 8/9 Solutions 1999

1. It is probably best to start by saying that we use 5 \$2 coins, or 4, or 3, or 2, or 1, or 0. If we use 5 \$2 coins, the game is over, so that gives 1 way of making change. If we use 4 \$2 coins, we need to produce 2 dollars with loonies and quarters. That can be done in 3 ways (2 loonies, or 1, or 0.) If we use 3 \$2 coins, the remaining \$4 can be done in 5 ways, and so on. So the number of ways of making change is $1 + 3 + 5 + 7 + 9 + 11$, that is, 36.

2. Let the weight of one baby be x in kilograms. Then, the total weight is given by $2x + 8x = 10x = 75\,\text{kg}$. Solving, we see that $x = 7.5\,\text{kg}$. Since the mother is 8 times as massive as a child, the mother is $8 \times 7.5 = 60\,\text{kg}$.

3. (a) 8; (b) 24; (c) 24

4. Let the cost of one adult's food ration be x. Then, the total cost of the food is $4x + 2.5x = 130$, or $x = 20$. Family A has the equivalent of 4 adults, so they have to pay \$80.

5. Turn both on. On the third beep of the five minute timer put the egg in, on the second beep of the nine minute timer take the egg out (there are other solutions). In general, if we have an a minute timer and a b minute timer, where a and b are integers with greatest common divisor d, then we can time a kd minute egg for any positive integer k.

6. \$2.20
There are many ways of solving the problem without "algebra." For example, it is clear that a drink costs 50 cents more than a popcorn. After buying 3 popcorns and 2 drinks, trade in the 2 drinks for 2 popcorns, getting a dollar back. So 5 popcorns cost \$11.
Or else note that 5 popcorns and 5 drinks cost \$24.50, so 2 of each cost \$9.80. Thus getting 1 more popcorn must cost \$2.20.

7. 21

For the sake of linguistic simplicity, the phrase "number of female ancestors" is used, rather than the more correct "maximum number of female ancestors." To put it another way, if someone is your great-grandmother through your mother's side and also your father's side, she is counted twice. Let F_n be the number of female ancestors n generations back, and M_n the number of male ancestors. So $F_0 = 1$ and $M_0 = 0$. We have $M_{n+1} = F_n$ and $F_{n+1} = M_n + F_n$. Thus $F_{n+1} = F_{n-1} + F_n$, so we get the familiar Fibonacci sequence.

Students can find the answer in this particular case by simply drawing the family tree. Some will find the recurrence relation obvious.

8. (a) Let V be the spider's initial position, and f any one of the three faces that V is on. Travel almost all the way around f (say counterclockwise) until you reach the vertex W adjacent to V. Travel on the edge through W perpendicular to f, then travel clockwise on the face parallel to f until you reach the edge adjacent to V, and finally return to V.

(b) The way described in part (a) is essentially the only way to meet the conditions. One of the directions will be traversed four times, the other two directions twice each. The length of the minimal path is $4(3) + 2(4 + 5)$, that is, 30.

9. 5000 Taiwan dollars at 3.724 Y each is 18620 Y. Thus, the change in yen is 2620 Y. Every 3.724 Y is equal to a Taiwanese dollar, so we have 2620/3.724 Taiwan dollars. It takes \$20.37 T to buy \$1 Cdn, so the change in Canadian dollars is $\dfrac{2620}{3.724 \times 20.37} \approx \$34.54$ Cdn.

10. $200\,\text{cm}^2$

Turn the inner square until its vertices meet the sides of the outer square. Draw the diagonals of the inner square. It is then obvious by dissection that the inner square has half the area of the outer square. We could instead use the Pythagorean Theorem, but there is no certainty that a grade 8 student will know it.

11. After Agnes has taken a seat, there are two seats out of nineteen that are beside her (to the left and to the right). Thus, the probability is 2/19.

12. There are $400/5 = 80$ numbers between 1 and 400 that *are* divisible by 5. Thus, there are 320 numbers between 1 and 400 that *aren't* divisible by 5.

13. Let the length of the side be x. Then, originally, the area A is $x^2 = A$. After the lengths have been increased, the new area is $(1.2x)^2 = 1.44x^2 = 1.44A$. Thus, the area has increased by 44%.

14. 99% of 600 is 594, so the pumpkin had 594 lbs of water originally. Let the weight of the water lost be x in pounds. Then, after sitting in the sun, the pumpkin has lost x lbs of water, and therefore x lbs of total weight. We also know that the pumpkin is now 98.5% water. Therefore, $\frac{594-x}{600-x} = 0.985$ Solving this, we see that $x = 200$ lbs, and therefore the total weight of the pumpkin is $600 - x = 400$ lbs.

15. Using the hint, we see that the two of them together earn \$366 each day. There are 365 days in a year, so in that year, Jo earns $(366 \cdot 365)/2$, that is, \$66795.

16. (a) We must be careful. There are two cases: the block can either have so much volume that it displaces the water so that it is totally submerged, or it can have less volume, and thus has a part sticking up out of the water line. Assume that for one block, that the cube does not end up being totally submerged. Let the height of the amount submerged be h in cm. Then, the amount of water displaced by the block is 25^2h and this is equal to the volume formed by the waterline and the tub minus the original volume of water, $5000 \times h - 5000 \times 20$. Therefore, $4375 \times h = 100000$ and $h = 160/7$. Notice that this is less than the side of the cube (25 cm), so our assumption was correct. The question asks for how much the water level rose. This is $160/7 - 20 = 20/7$, or about 2.857 cm.

(b) Again, assume that the blocks are not totally submerged. Then, the change in volume is $2 \times 25^2 \times h$ and this is equal to $5000^2 \times h - 50000 \times 20$. This time, h is larger than 25 cm, so the blocks are submerged completely. Therefore, we must recalculate with the assumption that the blocks are completely under water. Let the depth of the new water level be l. Then, l must be equal to the sum of the volumes of the original water mass and the new concrete masses, divided by the base area of the tub. $l = (100000 + 2 \times 25^3)/5000 = 105/4$. How much

did the water level rise? $105/4 - 20 = 25/4$. The question asks how much *more* the water rises after adding the second block... this is $25/4 - 22/7 = 97/28$, or about 3.393 cm.