UBC Grade 5-8 Solutions 1998

1. Divide the figure into a 9×12 rectangle and a triangle. The long leg of the triangle is 12, and the hypotenuse is 13. By the Pythagorean Theorem, the short leg of the triangle is $\sqrt{13^2 - 12^2} = 5$. The area of the triangle is thus $bh/2 = 12 \times 5/2 = 30$. The area of the rectangle is $12 \times 9 = 108$. Thus, the total area is $30 + 108 = 138\,\mathrm{m}^2$.

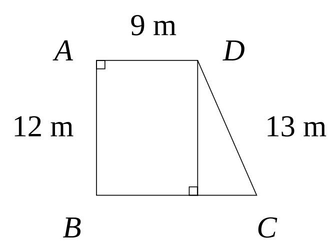

Figure 1: Dividing the Trapezoid

2. The distance between centres is $8\,\mathrm{cm}$; we can conclude that the radius is $4\,\mathrm{cm}$. The rectangle thus has a length of $4 \times 4 = 16$ and a width of $2 \times 4 = 8$. This results in an area of $8 \times 16 = 128\,\mathrm{cm}^2$.

3. Let the amount of juice added be x in L. Then, the original concentration is $\frac{100}{1000}$. The final concentration is

$$0.4 = \frac{100 + x}{1000 + x}$$

Solving this equation, we find that $x = 500\,\mathrm{mL}$.

4. Line up the guests. The first person must take a picture with 11 people, after which he or she may leave the line, as no one in the line will need to take another picture with that person. The second person needs to take a picture with 10 people. (The second person has already had a picture with the first.) After this, the second person may leave the line-up. The third person needs to take a picture with 9 people... etc... Thus a total of $11 + 10 + 9 + \cdots + 2 + 1 = 66$ pictures need to be taken.

5. Notice that with 4 darts, the maximum score attainable is 20. Thus, we must have more than 4 darts. Observe that $5 + 5 + 5 + 3 + 3 = 21$, and therefore 5 darts enable us to score 21 points.

6. June has 30 days. If June 1st is a Sunday, then the Sundays occur on the dates 1, 8, 15, 22, 29. If June 2nd is a Sunday, then the Sundays occur on the dates 2, 9, 16, 23, 30. If the first Sunday of June occurs any later, then we won't have 5 Sundays in June.
If June 1st is a Sunday, then June 8th is a Sunday, and June 11th is a Wednesday. If June 2nd is a Sunday, then June 9th is a Sunday and June 11th is a Tuesday.

7. Since it takes 9 hours to fill the pool to 3/5 capacity, it takes 3 hours to fill 1/5 of the capacity. We have 2/5 of the pool left to fill, which requires 6 additional hours.

8. We need to compare areas. Let x be the number of pieces that we choose to cut the pizza into. The radius of the 9" pizza is 4.5", and the radius of the 12" pizza is 6". We need $\frac{6^2\pi}{x} \geq \frac{4.5^2\pi}{8}$, therefore, $x \leq 14.2222...$ We need a whole number for the number of pieces, so if we cut the 12" pizza into 14 pieces, we can have slices at least as big as those for the 9" one.

9. The probability that Vancouver wins the series is $1 -$ P(Edmonton wins both remaining games). Since the teams are evenly matched, the probability that Edmonton wins a particular game is 0.5. Therefore, the probability that Edmonton wins 2 out of 2 games is $0.5 \times 0.5 = 0.25$, and the probability that Vancouver wins the series is thus $1 - 0.25 = 0.75$.

10. $10 bill - 4 toonies, 1 loonie, 3 quarters, 4 dimes, 4 pennies = $10.19
$5 bill - an infinite number of toonies

11. Pretend that we have a lot of tables piled next to each other, so that the ball can travel as shown (see next page). Then, we know that the ball will enter a corner pocket when the vertical distance travelled is a multiple of 6 and 10, since the pool table is 10 by 6. This happens at 30 feets' worth of vertical movement. This corresponds to 3 isosceles

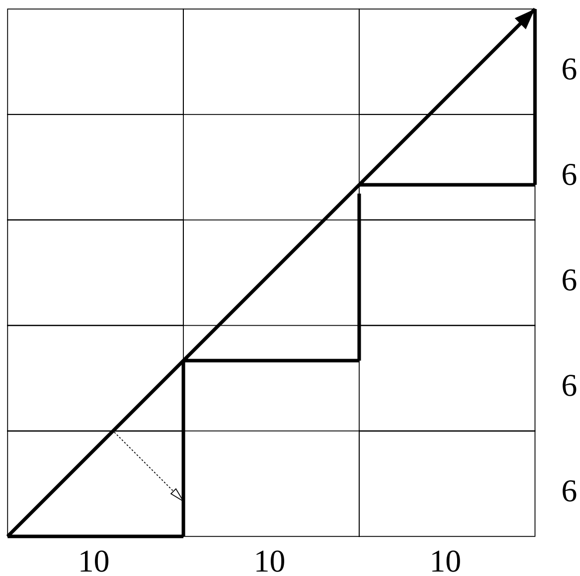

Figure 2: Virtual Pool Table

right triangles with side length 10 and hypotenuse $10\sqrt{2}$. Since the ball travels along the hypotenuses, it has travelled $3(10\sqrt{2}) = 30\sqrt{2}$ feet.

12. average speed = total distance travelled / total time elapsed
We don't know the route length – let us denote this length as 1 "unit", where the unit in question is a length measured in kilometers. Then, the total transit time is $\frac{1}{20} + \frac{1}{30}$, and the total distance travelled is 2. The average speed is

$$\frac{2}{\dfrac{1}{20} + \dfrac{1}{30}} = 24\,\text{kmph}$$

If the return trip is extended, let us denote the new route length by $1 + d$. Then, the average speed would be

$$\frac{2+d}{\dfrac{1}{20} + \dfrac{1+d}{30}} = \frac{2+d}{\dfrac{30+20+20d}{600}} = \frac{120+60d}{5+2d}$$

If we let d get really small, then this is close to the case before, an average speed of 24 kmph. However, if we let d get really large, then adding a small amount to it will not make a large relative effect – in this case, the speed approaches a value of 30 kmph. However, these are only limits of the speeds that are possible (we can get as close to them as we wish by choosing appropriate values of d), and the average speed is thus between 24 and 30 kmph.

13. The floor can be divided into squares of side length 10 cm each. $7\,\text{m} = 700\,\text{cm}$ and $5 = 500\,\text{m}$, so the floor is 70 tiles by 50 tiles, for a total of $70 \times 50 = 3500$ tiles. Each box contains 30 tiles, and when we divide $3500/30$, we get a decimal between 116 and 117. Hence, we need to purchase 117 boxes of tiles to adequately cover the floor.

14. Note that $27 = 3^3$, so the price must be divisible by 9. Let the smudged digits be represented by x and y. Then, $x.5y$ must be divisible by 9. The sum of the digits is divisible by 9 iff the number is divisible by 9. Therefore, $x+y+5$ is either of 9 or 18 – higher numbers aren't possible because x and y are *single*-digit numbers.
If $x + y = 4$, then we can have $(x, y) = (0, 4), (1, 3), (2, 2), (3, 1), (4, 0)$. Checking each of these, we see that 27 divides \$0.54 and \$3.51.
Checking the rest of the cases ($x + y = 13$), we see that \$0.54, \$3.51, \$4.59, and \$7.56 are divisible by 27, corresponding to an individual cherry tomato plant cost of \$0.02, \$0.13, \$0.17, and \$0.28.

15. In Blaine, 10 gallons of gas at \$1.30 US is worth \$13 US. Since each US dollar is worth \$1.40 Cdn, filling up in Blaine costs $13 \times 1.4 =$ \$18.20 Cdn. 10 gallons is equivalent to 38 L. We are told that gas in Vancouver is 60 Canadian cents per litre, so we would have to spend $38 \times 0.6 =$ \$22.80 Cdn for a full tank of gas in Vancouver. Subtracting, we find that we save \$4.60 Cdn by filling up in Blaine. Note, however, that you would have to drive to and from Blaine, thus spending some of the money you save.

16. The product of the faces is either even or odd, and if it's odd, then all the die faces must be odd, because only odd numbers multiply to odd numbers. Therefore, P(5 faces odd) $= 1/2^5 = 1/32$ and P(product is even) $= 1 - 1/32 = 31/32$.

17. Observe that once we have found 13 integers in a row that can be written as the sum of multiples of 7 and 13, then all subsequent numbers can be written as the sum of multiples of 7 and 13. Let a be the last of these 13 integers. Then, $a+1$ can be written as $a-13+2(7)$. $a+2$ can be written as $a-2(13)+4(7)$. $a+3$ can be written as $a-3(13)+6(7)$, etc. Now we need to list positive combinations of 7 and 13:

List the multiples of 7:

7, 14, 21, 28, 35, 42, 49, 56, 63, 70, 77, 84, 91, ...

List the multiples of 13:

13, 26, 39, 52, 65, 78, 91, 104, ...

Each of the next lines is the result of repeatedly adding 7 to the multiples of 13:

20, 33, 46, 59, 72, 85, 98, ...

27, 40, 53, 66, 79, 92, ...

34, 47, 60, 73, 86, ...

41, 54, 67, 80, 93, ...

48, 61, 74, 87, ...

55, 68, 81, ...

62, 75, ...

69, 82, ...

76, ...

83, ...

Putting these in order yields:

7, 13, 14, 21, 26, 27, 28, 33, 34, 35, 39, 40, 41, 42, 46, 48, 52, 53, 54, 55, 56, 59, 60, 61, 62, 63, 65, 66, 67, 68, 69, 70, 72, 73, 74, 75, 76, 77, 78, 79, 80, 81, 82, 83, 84, ...

We now have 13 numbers in a row that can be written as positive combinations of 7 and 13. Therefore, the highest number of doughnuts that can't be bought using only boxes of 7 and 13 is 71.