UBC Grade 11/12 Problems 1997

1. Evaluate $(1-\frac{1}{2})(1-\frac{1}{3})\cdots(1-\frac{1}{1996})(1-\frac{1}{1997})$.

2. Solve for $x : x = \sqrt{6+\sqrt{6+\sqrt{6+\cdots}}}$

3. Among grandfather's papers, the following bill was found:

27 cherry tomato plants $.5**

The first and last digits were so smudged as to be illegible. What are the possible prices of one cherry tomato plant?

4. It takes Alex three hours to row down a river from point A to point B, and four hours to row up the river from B to A. How long would it take for a piece of wood to drift from A to B?

5. The cost of living in January rose by 0.3%. What annual inflation rate does this correspond to?

6. The following letter (true story) was received by a professor at UBC from Sandspit, BC in June 1992:

I have a small problem which has been plaguing me and my fellow builders for some time now.

What we are seeking is a mathematical equation for determining rafter angles. I came up with what I thought was a good formula but it does not quite seem to work. I began with a roof having a pitch of 4/11. Assuming, of course, that the right angle is 90 degrees, this means that the other two angles must add up to 90 degrees, I added 4 and 11 together obtaining 15. I then divided 90 by 15, giving me 6. From there, I multiplied 6×11 to give me 66 degrees as one angle and 6×4 to give me 24 degrees as the other angle.

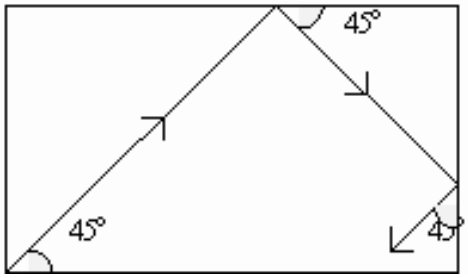
Now, although we thought we were on the right track here we found that the rafters were always out by a few degrees. We have not ruled

out the possibility that we made a mistake in measurement somewhere but before we go through the whole process again, perhaps you could tell us if the equation is correct or not. If it is not, please put us on to the correct equation (try to keep it fairly simple as we have not been in school for quite a few years and our math skills are a little rusty).

Thank you.

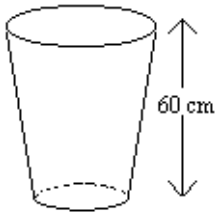
Is the answer given by the builders correct? If not, what is the correct answer to the nearest degree?

7. A billiard ball is rolled from the corner of a 6 ft $\times$ 10 ft billiard table and it continually rolls off each wall at an angle of 45 degrees. Does the ball eventually land at a corner pocket? If it does, how far does the ball travel?



8. A regular hexagon has sides of length 4 cm. Find the area of the triangle formed by connecting alternate vertices of the hexagon.

9. Consider an open garbage can in the shape of a frustum of a cone (truncated cone). The diameter of the open top is 40 cm, the diameter of the bottom is 30 cm and the height of the garbage can is 60 cm. Suppose the garbage can collects 10 cm of rain during a rainstorm. Determine the amount of rain which fell during the rainstorm. [The volume of a cone of base radius a and height h is $\pi a^2 h/3$.]



10. A missile is launched from a launch pad at an angle of 30 degrees to the ground and then moves along a straight line path. It passes 1 km directly above Jo's house.
(a) How far is the launch pad from Jo's house?
(b) Suppose a laser beam emanating from Jo's house illuminates the missile when it is 2 km away. At what angle (to the nearest degree) is the laser beam aimed?

11. What is the area of the set of points (x, y) given by the inequalities $|2x - 3y| \leq 12$ and $|2x + 3y| \leq 12$?

12. Let x and y be arbitrary real numbers. Find the minimum values of the expressions:
(a) $2x^2 + 3y^2 - 4x - 5y + 1$; (b) $x^2 - xy + y^2 + y$.

13. Five dice (six-sided, numbered 1-6) are rolled and their top faces are examined. Find the probability that:
(a) the sum of the faces is even;
(b) the product of the faces is even.

14. Which numbers divide $(n + 1)^3 - (n + 1)$ for every positive integer n?

15. (a) Find all values of x for which $1 - 3x \geq (1 - x)^3$.
(b) Find all values of p for which $1 - px \geq (1 - x)^3$ for any $x \geq 0$.