UBC Grade 10 Problems 1997

1. An Eastjet plane flies roundtrip from Vancouver to Kelowna. With a head wind, its speed is 240 km/h; with a tail wind its speed is 360 km/h. What is the plane's average speed for the roundtrip?

2. Evaluate: $(1-\frac{1}{2})(1-\frac{1}{3})\cdots(1-\frac{1}{1996})(1-\frac{1}{1997})$.

3. Among grandfather's papers, the following bill was found:

27 cherry tomato plants $.5**

The first and last digits were so smudged as to be illegible. What are the possible prices of one cherry tomato plant?

4. After playing 30 games in goal, Captain Birk has a Goals Against Average (GAA) of 3.00. [GAA = (the number of goals allowed)/(the number of games played).] For the rest of the season, Captain Birk has five shutouts (games in which no goals are allowed) and finished with a GAA of 2.00 for the whole season. What is the minimum number of games in goal played by Captain Birk during the season?

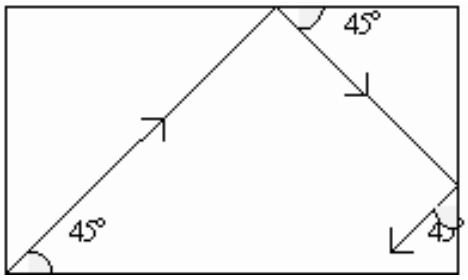
5. It takes Alex three hours to row down a river from point A to point B, and four hours to row up the river from B to A. How long would it take for a piece of wood to drift from A to B?

6. January 1st, 1997 fell on a Wednesday. The first day of the twentieth century (January 1st, *1901*), fell on a...?

7. Students who passed the last Honours Math 10 exam like mathematics. Which of the following statement(s) must be true? Which must be false?
(a) All students who like mathematics passed the last Honours Math 10 exam.
(b) All students who dislike mathematics failed the last Honours Math 10 exam.
(c) Some students who dislike mathematics passed the last Honour Math 10 exam.
(d) Some students who dislike mathematics failed the last Honours Math 10 exam.
(e) Some students who like mathematics passed the last Honours Math 10 exam.

8. Barbara cycles to school alongside a railroad track at 6 km/h. Every day she arrives at a crossing at the same time as the daily train. One day she is 50 minutes later than usual in arriving at the crossing. Consequently, the train, which is always on time, passes Barbara 6 km before the crossing. What is the speed of the train?

9. A billiard ball is rolled from the corner of a 6 ft by 10 ft billiard table and it continually rolls off each wall at an angle of 45°. Does the ball eventually land at a corner pocket? If it does, how far does the ball travel?



10. David visits his friend Nicole and then returns home by the same route. Along his route David goes uphill, downhill, and on level ground. David walks 2 km/h when going uphill and 6 km/h when going downhill. Suppose his total walking time for the roundtrip is six hours and the total distance he walks is 18 km. How fast does David walk on level ground?

11. In how many ways can a careless secretary place two letters in two envelopes so that no one gets the right letter? What about the same problem with two replaced by three? Two replaced by four? Two replaced by seven?

12. Find all integers $n \geq 0$ for which $n^2 - n + 2$ is a prime number.

13. Let $[x]$ denote the greatest integer less than or equal to x. For example, $[3] = 3$, $[5.7] = 5$. Find all integers n satisfying the equation $[\sqrt[3]{1}] + [\sqrt[3]{2}] + \cdots + [\sqrt[3]{n}] = 2n$.

14. Five dice (six-sided, numbered 1-6) are rolled and their top faces are examined. Find the probability that:
(a) the sum of the faces is even.
(b) the product of the faces is even.

15. Which numbers divide $(n+1)^3 - (n+1)$ for every positive integer n?