UBC Grade 5-8 Solutions 1997

1. The total of all the marks before the error was spotted was:
number of students $\times$ average mark $= 18 \times 86$.
After the error was found, the total is lowered by the error amount, $86-68 = 18$. Thus, the new, correct total is $18\times86-18 = 18\times(86-1) = 18 \times 85$, and therefore the correct average is $18 \times 85/18 = 85$.
A quicker way of finding the answer would be to take note of the fact that the sum of the marks went down by 18, so that the average must have gone down by $18/18 = 1$.

2. The conversion factor is 7.5 mm rain per 8 cm snow, or 7.5 mm rain per 80 mm snow. Thus, the answer we seek is $500 \times (7.5 \ / \ 80) = 46.875$ mm of rain.

3. The maximum number of games played in a series is 7 (at this point, one team must have won 4 games). The loser of the series is eliminated and plays no more games. Thus, in the first round, there are 8 series; in the second, 4; in the third, 2; and, finally, in the fourth, 1. Since each series has a maximum of 7 games, the maximum number of playoff games is:

$$8 \times 7 + 4 \times 7 + 2 \times 7 + 1 \times 7 = 15 \times 7 = 105$$

An alternate way of viewing the problem is recognizing that 15 teams need to be eliminated, since there can only be one winner out of 16. Since each elimination requires the playing of a series, we need to 15 series, at a maximum of 7 games each, for $7\times15 = 105$ games maximum.

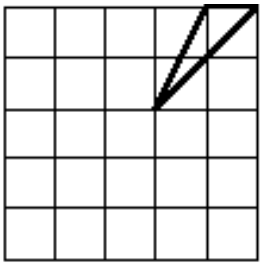
4. Let $c = \#$ of cars, $b = \#$ of bicycles, $p = \#$ of people. Then we are given 3 equations, namely:

$$\begin{array}{lrcrcrl} (1) & 500c & + & 400b & & & = d \\ (2) & 600c & & & & & = d \\ (3) & & & 400b & + & 3000p & = d \end{array}$$

where d represents the capacity of the ferry. Then, from the first and third equations, we see that a car is worth six passengers, i.e. $c = 6p$. We can use this result with equation (2) to find that $600 \times 6p = d$, or in other words, the ferry can carry 3600 people.

5. Let $x =$ the number of goals let in after the first 30 games. We know that the number of goals let in during the first 30 games was 90, and that the total number of games played is at least 35. We are given that the final GAA was 2. The GAA has gone down, and we are asked to find out the minimum possible number of games played. The fastest way that the GAA can decrease is if no goals are let in at all for the remainder of the season, but we are told that there were only 5 shutouts. $90/35 \approx 2.57$, which is too big. The next fastest way of decreasing the GAA is to let in one goal per game. Let the additional number of goals scored after the first 90 be x. Then, we need $\frac{90+x}{35+x} = 2.00$ which we can simplify as $x = 20$. Thus, the minimum total games played during the season is $35 + 20 = 55$.
Note that we treated the GAAs as exact in the solution to this problem. What if they had actually been rounded? It turns out that even if the GAAs had been rounded, there would have been no confusion as to how many goals were scored or how many games were played, because the errors involved are small enough.

6. Notice that the figure is a triangle, and recall that the area of a triangle is $bh/2$. Let the base be the topmost side; then the height is 2 and the base is 1. The area of the triangle is thus 1 square unit. The total area is $5 \times 5 = 25$ square units; this implies that the region just outside the triangle has $24/25$ or 96% of the area.



7. At every least common multiple of their departure intervals, the buses embark simultaneously from Granville and Georgia. Since the least common multiple of 12 and 20 is 60, the buses depart at the same time every hour on the hour. There are 17 "on the hours" between 7 a.m. and 11:30 p.m.

8. The ball is released from a height of 16 m, so it travels first a distance of 16 m. Then it proceeds to traverse each of the distances of 8, 4, and 2 m twice each. Finally, it travels a distance of 1 m only once. The total distance travelled is

$$16 + (2 \times 8) + (2 \times 4) + (2 \times 2) + 1 = 45\,\text{m}.$$

9. Cubes inside the first layer have no faces painted red. Cubes on the outmost layer have 1 face painted red only if they are not on an edge. Only the 4 cubes in the centre of each face satisfy this requirement. Thus, there are $6 \times 4 = 24$ cubes with only one face painted red.

10. (a) 32% of the decided voters (75% of the population) were to vote Liberal. Conceivably, a range 0% to 100% of the undecided (25% of the population) will vote Liberal. If 0% of the undecideds vote Liberal, then the total percentage of people who vote Liberal would be *estimated* (this is a poll!) to be 32% of 75% $= \dfrac{32}{100} \times \dfrac{75}{100} = 24/100$ or 24%.

If all of the undecideds vote Liberal, then the total percentage of people who vote Liberal would be estimated to be $\dfrac{32}{100} \times \dfrac{75}{100} + \dfrac{100}{100} \times \dfrac{25}{100} =$ 49%.

Thus, the we would guess, based on the poll, that the Liberals will get between 24 and 49 percent of the votes.

(b) If the Liberals received 32% of the vote, then they must have received 32% of the "undecided" votes (assuming that the people polled did not lie), because 32% of the decideds were to vote Liberal. Thus, the NDP could have received 0 to 68 percent of the undecided votes. This corresponds to a percentage range of $44\% \times 75\%$ to $44\% \times 75\% + 68\% \times 25\%$], or, simplified, 33 to 50 percent of the votes.

11. The CDs were at least \$15, therefore the CDs must have cost at least $38 \times \$15 = \$570$. Let the missing digits be denoted by x and y, that is,

let the total purchase price be $x29.2y$, where x and y are placeholders for *single*-digit numbers. The minimum total price calculated shows us that x must be one of 6, 7, 8, 9. We also know that the prices for each of the CDs was the same; thus, $x29.2y$ must be divisible by 38 (and thus must be divisible by 2, implying that y must be even). Trying each combination of valid x and y, we see that the only combinations that are divisible by 38 are \$629.28, and \$729.22. These results correspond to an individual CD price of \$16.56 or \$19.19.

12. The answer is (b) and (d).

13. There is one "way" to get to the top of the topmost intersection. The next highest intersections can be reached in only one way each (take the paths left or right form the top). In fact, it can quickly be seen that the number of ways to reach any intersection on the diagram is the sum of the number of ways to get to the two intersections immediately above it:

1
1 1
1 2 1
1 3 3 1
1 4 6 4 1
1 5 10 10 5 1
1 6 15 20 15 6 1

This triangle is known as Pascal's Triangle. It is constructed using the summing rule mentioned above. The lowest point on the given diagram corresponds to the number 20 on Pascal's Triangle, so we conclude that there are 20 ways to get to the bottom of the square if we are only allowed to move downward.

14. The train passes Barbara 6 km before the crossing. This corresponds to 1 hour before Barbara reaches the crossing. 1 km later, corresponding to 10 minutes, Barbara would have reached the crossing had she not been late – at this time, the train has reached the crossing. Therefore, in 10 minutes = (1/6) hour, the train travelled 6 km. Therefore, the train moves at a speed of 36 kmph.