UBC Grade 11/12 Problems 1995

1. Solve for $x : 5x - 6 \geq x^2$.

2. Solve the inequality: $|3x| \geq |6 - 3x|$.

3. Solve the equation: $\log_1(\log_2(\log_3(\log_4 x))) = 0$.

4. For which values of α does the equation $x^2 + x + \alpha = 0$ have:
(i) no real roots?
(ii) one real root?
(iii) two real roots?
(iv) more than two real roots?

5. A Lime Air plane flies round trip from Vancouver to Kelowna return. With a head wind its speed is 240 km/h; with a tail wind its speed is 360 km/h. What is its average speed for the round trip?

6. For a given principal P, what annual interest rate yields the same amount of income as 8 1/8 % compounded monthly? [As of December 1st, 1994, Canada Trust offered 8 1/4 % annual interest or 8 1/8 % compounded monthly.]

7. Find the equation of a circle having $(5, -6)$ and $(-1, 4)$ as ends of a diameter.

8. How many solutions has the equation $\sin x = x/30$?

9. Find the number of real solutions of the equation $x^5 + x^3 - 8x^2 - 8 = 0$.

10. Find all integers $n \geq 0$ for which $n^2 - n + 2$ is a prime number.

11. Let $[x]$ denote the greatest integer less than or equal to x. For example, $[3] = 3$, $[5.7] = 5$. If $[\sqrt[3]{1}] + [\sqrt[3]{2}] + \cdots + [\sqrt[3]{n}] = 2n$, find n.

12. January 1st, 1995 fell on a Sunday. The first day of the twentieth century (January 1st, *1901*), fell on a...?

13. A deck of 16 cards contains four aces, four kings, four queens, and four jacks. The 16 cards are thoroughly shuffled and your opponent Richard Hickson (who always tells the truth) draws two cards simultaneously and at random from the deck. He says, "I hold at least one ace." Find the chance that he holds at least two aces in his hand.

14. Fifteen billiard balls are lying on a pool table in such a way that they are just squeezed inside an equilateral triangular frame whose inside perimeter is 876 mm. Find the radius of a billiard ball.

15. Consider $\triangle ABC$. Let p be a point interior to $\triangle ABC$ and let P be a point on the boundary of $\triangle ABC$. Let $L(p, P)$ be the distance from p to P. Let S be the perimeter of $\triangle ABC$.
(i) Prove that $L(p, P) < S/2$
(ii) Find a triangle for which the inequality $L(p, P) < 0.49999S$ is false for some (p, P).