UBC Grade 5-8 Solutions 1995

1. Divide the figure into a 9×12 rectangle and a triangle. The long leg of the triangle is 12, and the hypotenuse is 13. By the Pythagorean Theorem, the short leg of the triangle is $\sqrt{13^2 - 12^2} = 5$. The area of the triangle is thus $bh/2 = 12 \times 5/2 = 30$. The area of the rectangle is $12 \times 9 = 108$. Thus, the total area is $30 + 108 = 138\,\mathrm{m}^2$.

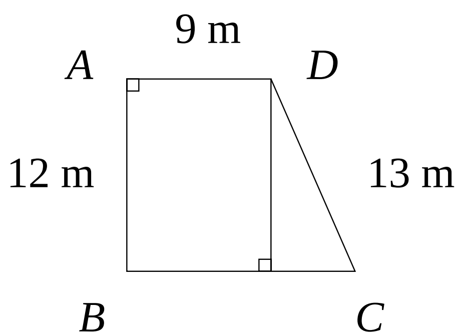

Figure 1: Areas

2. The distance between centres is 8 cm; we can conclude that the radius is 4 cm. The rectangle thus has a length of $4 \times 4 = 16$ and a width of $2 \times 4 = 8$. This results in an area of $8 \times 16 = 128\,\mathrm{cm}^2$.

3. Line up the guests. The first person must take a picture with 11 people, after which he or she may leave the line, as no one in the line will need to take another picture with that person. The second person needs to take a picture with 10 people. (The second person has already had a picture with the first.) After this, the second person may leave the line-up. The third person needs to take a picture with 9 people... etc... Thus a total of $11 + 10 + 9 + \cdots + 2 + 1 = 66$ pictures need to be taken.

4. The string "TALMUD TORAH" has 12 letters. Thus, 38 blank spaces are available for spaces in total; to centre, we must leave 19 blanks on each side.

5. Notice that with 4 darts, the maximum score attainable is 20. Thus, we must have more than 4 darts. Observe that $5 + 5 + 5 + 3 + 3 = 21$, and therefore 5 darts enable us to score 21 points.

6. June has 30 days. If June 1st is a Sunday, then the Sundays occur on the dates 1, 8, 15, 22, 29. If June 2nd is a Sunday, then the Sundays occur on the dates 2, 9, 16, 23, 30. If the first Sunday of June occurs any later, then we won't have 5 Sundays in June.
If June 1st is a Sunday, then June 8th is a Sunday, and June 11th is a Wednesday. If June 2nd is a Sunday, then June 9th is a Sunday and June 11th is a Tuesday.

7. Since it takes 9 hours to fill the pool to 3/5 capacity, it takes 3 hours to fill 1/5 of the capacity. We have 2/5 of the pool left to fill, which requires 6 additional hours.

8. 20 goals were let in in 5 games. If the worst game is removed, $4 \times 2 = 8$ goals were scored. Thus, $20 - 8 = 12$ goals were scored against him in his worst game.

9. average speed = total distance travelled / total time elapsed
We don't know the route length – let us denote this length as 1 "unit", where the unit in question is a length measured in kilometers. Then, the total transit time is $\frac{1}{20} + \frac{1}{30}$, and the total distance travelled is 2. The average speed is

$$\frac{2}{\dfrac{1}{20} + \dfrac{1}{30}} = 24\,\text{kmph}$$

If the return trip is extended, let us denote the new route length by $1 + d$. Then, the average speed would be

$$\frac{2+d}{\dfrac{1}{20} + \dfrac{1+d}{30}} = \frac{2+d}{\dfrac{30+20+20d}{600}} = \frac{120+60d}{5+2d}$$

If we let d get really small, then this is close to the case before, an average speed of 24 kmph. However, if we let d get really large, then adding a small amount to it will not make a large relative effect – in this case, the speed approaches a value of 30kmph. However, these are only limits of the speeds that are possible (we can get as close to them as we wish by choosing appropriate values of d), and the average speed is thus between 24 and 30 kmph.

10. Each square meter contains $10^2 = 100$ square decimeters. We have a floor of $5 \times 7 = 35$ square meters, translating into 3500 square decimeters or 3500 tiles. When we divide 3500 by 30, we get a number between, 116 and 117, therefore we need to buy 117 boxes to adequately cover the floor.

11. $n^2 - n + 2 = n(n-1) + 2$. Since $n(n-1)+2$ is always even, and the only even prime is 2, we need $n(n-1) = 0$, meaning that $n = 0, 1$.

12. In Blaine, 10 gallons of gas at \$1.10 US per gallon is worth \$11 US. Since each US dollar with worth \$1.40 Cdn, a full tank of gas in Blaine costs $11 \times 1.40 =$ \$15.40 Cdn.
There are 3.8 L per gallon, so a full tank of gas (10 gallons) is equivalent to 38 L. Each litre of gas in Vancouver costs \$0.60 Cdn, so we must spend $38 \times 0.60 =$ \$22.80 Cdn to fill up in Vancouver.
Subtracting, we find that we save \$7.40 Cdn filling up in Blaine. Note, however, that you would have to drive to and from Blaine, thus spending some of the money you save.

13. Observe that once we have found 7 positive integers in a row (all at least 25) that can be written as the sum of positive multiples of 7, 13, and 25, then all subsequent integers can be written as the sum of positive multiples of 7, 13, and 25. Let a be the last of these 7 integers in a row. Then, $a+1 = (a-25) + 2(13), a+2 = (a-2(13)) + 4(7), a+3 = (a-25) + 4(7)$, etc... It turns out that we can keep subtracting and rewriting because the smallest of the 7 integers in a row is at least 25. List the first 100 integers or so (we will only list the first 60, because we know the answer from hindsight), and then we can start eliminating combinations of multiples of 7, 13, and 25 in a systematic way.

1	2	3	4	5	6	7	8	9	10
11	12	13	14	15	16	17	18	19	20
21	22	23	24	25	26	27	28	29	30
31	32	33	34	35	36	37	38	39	40
41	42	43	44	45	46	47	48	49	50
51	52	53	54	55	56	57	58	59	60

First, remove all the multiples of 7. Then, remove all the numbers that are a multiple of seven plus a multiple of 13 (we can do this systematically by removing numbers that are 13 more than multiples of 7, then numbers that are 26 more, etc). Then, remove all the numbers that are a multiple of 7 plus a multiple of 25. Our resulting table of numbers looks like this:

1	2	3	4	5	6		8	9	10
11	12	13		15	16	17	18	19	
	22	23	24	25	26			29	30
31					36	37	38		
		43	44	45					50
51	52						58		

Now, remove the multiples of 13, the multiples of 25, and the numbers that are combinations of multiples of 13 and 25:

1	2	3	4	5	6		8	9	10
11	12			15	16	17	18	19	
	22	23	24					29	30
31					36	37	38		
		43	44	45					
							58		

Observe that we now have 7 integers in a row (46 to 52), all at least 25, that can be written as the sum of multiples of 7, 13, and 25. We can quickly see that $45 = 7 + 13 + 25$, and that 44 cannot be written as a multiple of 7, 13, and 25, so that 44 is the largest number of doughnuts that can't be purchased.