

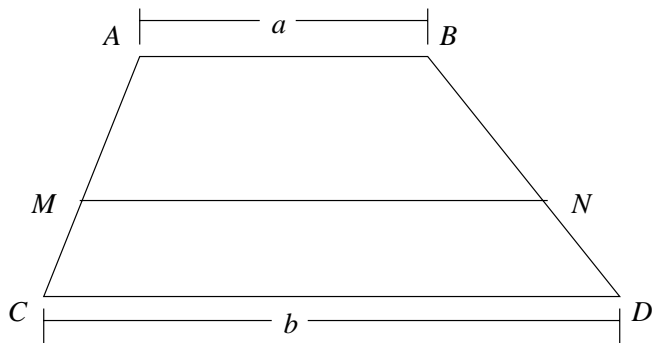
## UBC Grade 11/12 Problems 1994

1. Solve for  $x$  :  $x^2 \leq 5x - 6$ .
2. Solve the inequality:  $|3x| \geq |6 - 3x|$ .
3. Simplify:  $S = \frac{1}{\log_2 2} + \frac{1}{\log_3 2} + \cdots + \frac{1}{\log_{25} 2}$ .
4. A Lime Air plane flies roundtrip from Vancouver to Kelowna. The speed with a head wind is 240 km/h while the speed with a tail wind is 360 km/h. What is the average speed for the roundtrip?
5. For a given principal  $P$ , what annual interest rate compounded annually gives the same amount of income as 10% compounded semi-annually?
6. Find the equation of a circle having  $(5, -6)$  and  $(-1, 4)$  as ends of a diameter.
7. How many real solutions has the equation  $\frac{x}{100} = \sin x$ ?
8. Find the number of distinct real numbers satisfying the equation

$$x^3 + x - 8 = \frac{8}{x^2}$$

9.  $[x]$  denotes the greatest integer less than or equal to  $x$ . For example,  $[3] = 3$ ,  $[5.7] = 5$ . If  $[\sqrt[3]{1}] + [\sqrt[3]{2}] + \cdots + [\sqrt[3]{n}] = 2n$ , find  $n$ .
10. January 1st, 1994 fell on a Saturday. The first day of the twentieth century (January 1st, 1901), fell on a...?
11. A deck of 16 cards contains four aces, four kings, four queens, and four jacks. The sixteen cards are thoroughly shuffled and your opponent Richard Hickson (who always tells the truth) draws two cards simultaneously and at random from the deck. He says, "I hold at least one ace." Find the chance that he holds at least two aces in his hand.

12. In the figure,  $ABCD$  is a trapezoid, and  $MN$  is parallel to  $AB$  and  $DC$ . If  $MN$  bisects the area enclosed by  $ABCD$ , find the length of  $MN$ .



13. Fifteen billiard balls are lying on a pool table in such a way that they are just squeezed inside an equilateral triangle frame whose inside perimeter is 876 mm. Find the radius of a billiard ball.
14. A rectangular coordinate system has axes with scale 1 cm as the unit of length.  $\triangle PQR$  has vertices  $P = (0, 3)$ ,  $Q = (4, 0)$ ,  $R = (k, 5)$ , where  $0 < k < 4$ . If the area of this triangle is  $8 \text{ cm}^2$ , find the value of  $k$ .