UBC Grade 11/12 Problems 1993

1. At a price of \$1.00, Voodvord's can sell 1000 blank tapes which cost the store 60 cents each. For each penny that Voodvord's lowers the price, it can increase the number sold by 50. What price will maximise the profit?

2. $f(x) = 2x^3 - 1$ with domain of $f = (-\infty, \infty), g(x) = 3x + 1$ with domain of $g = (-\infty, \infty)$. Determine:
(a) $\frac{f(1)}{g(1)}$
(b) $f(g(1))$
(c) the inverse function of $f(x)$

3. Find the positive real values of x such that $x^{(x^x)} = (x^x)^x$.

4. At what time after 4:00 will the minute hand overtake the hour hand?

5. Find all values of m for which the straight line $y = x + m$ meets the ellipse $\frac{x^2}{16} + \frac{y^2}{9} = 1$ at
(a) one point
(b) two points
(c) no points.
Sketch the graphs when the two curves meet at one point.

6. If $y = 2x + |2 - x|$ express x as a function of y.

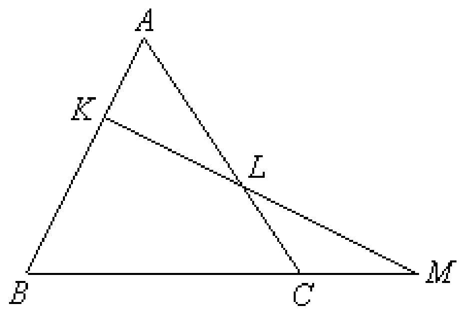
7. Let a, b and c be the lengths of the sides of a triangle. Show that if $a^2 + b^2 + c^2 = bc + ca + ab$, the triangle is equilateral.

8. Find the sum of the series $\frac{1}{2} + (\frac{1}{3} + \frac{2}{3}) + (\frac{1}{4} + \frac{2}{4} + \frac{3}{4}) + (\frac{1}{5} + \frac{2}{5} + \frac{3}{5} + \frac{4}{5}) + \cdots + (\frac{1}{100} + \cdots + \frac{99}{100})$.

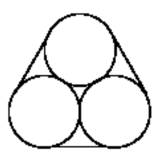
9. In how many ways can a careless secretary place four letters in four envelopes so that no one gets the right letter?

10. The remainder on dividing x^{100} by $x^2 + 3x + 2$ is a polynomial $P(x)$ of degree less than two. Find the constant term of $P(x)$.

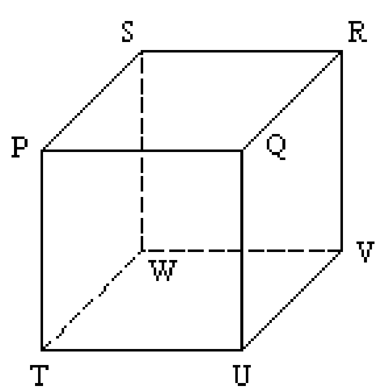
11. In the figure, $AB = AC$ and $KL = LM$. Find the ratio $\frac{KB}{LC}$.



12. Three pipes of diameter one meter are held together by a taut metal band as shown. Find the length of the metal band.



13. Given the cube $PQRSTUVW$ as shown, the plane which passes through P and the centres of the faces $TUVW$ and $UQRV$ intersects UV at X. Find the ratio $\frac{UX}{XV}$.



14. One is given real numbers a and b. Find all real numbers x such that

$$\frac{a\sin(x)+b}{b\cos(x)+a}=\frac{a\cos(x)+b}{b\sin(x)+a}$$

15. Without using calculus, find the area of the region bounded by

$$y=0, y=\sin^2(x), x=0, x=\pi/2$$

16. Let $\tan(\alpha)=1/m, \tan(\beta)=1/n$. Find all integers $m\geq 1, n\geq 1$, such that
(a) $\pi=4(\alpha+\beta)$
(b) $\pi=4(2\alpha+\beta)$
(c) $\pi=8(\alpha+\beta)$