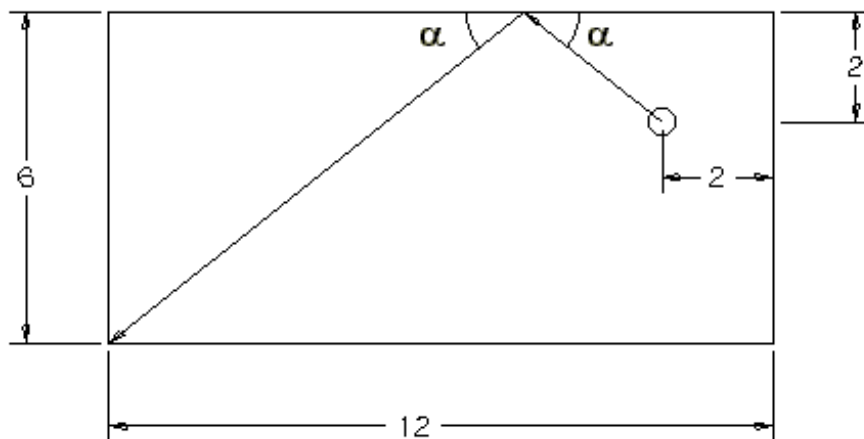
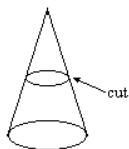


UBC Grade 11/12 Problems 1991

1. Find the area of a square inscribed in a circle of radius 1 cm.
2. When is the average of n consecutive integers an integer?
3. A billiard ball rests on a 6 ft by 12 ft pool table at the position shown in the diagram. A player wants to send it into a pocket along the indicated path. Find angle α .



4. A cone with a volume of 24 cm^3 is cut in two half-way up its axis. Find the volume of the lower part.



5. A farmer wants to enclose a rectangular field of area $A \text{ m}^2$ using a stone wall of length $x \text{ m}$ for one side and a total of $B \text{ m}$ of fencing for the other three. When does the problem have a solution? Is the solution unique?

6. Suppose a triangle has side lengths a, b , and c , with $a^2 + b^2 > c^2$. What can one say about the angle opposite side length c ?
7. In 1990-91, 83% of all first year students at UBC took a calculus course; one-half of all first year students were male; 43% of first year students in calculus courses were female. What percentage of first year male students took a calculus course in 1990-91? What percentage of first year female students took a calculus course in 1990-91?
8. John invests his money at an interest rate of 10% per annum. Margaret invests her money at the same interest rate compounded semi-annually. After one year, compare the amount of money earned by these siblings.
9. Find all m such that $mx - 1 + \frac{1}{x} \geq 0$ if $x > 0$.
10. Determine the number of roots of the equation $x^2 - 9 \sin x = 0$. Find all roots to two decimals.
11. Suppose $\log_{10} y = 3 + 5 \log_{10} x$. Sketch graphs of:
 - (a) $\log_{10} y$ vs. $\log_{10} x$
 - (b) $\log_5 y$ vs. $\log_5 x$
12. One is interested in the way the largest root r of the polynomial $x^2 - x + \alpha$ depends on parameter α . Sketch the graph of r as a function of α . Do the same for the polynomial $x^3 - x + \alpha$.
13. Find all x such that $|x - a| = N|x - b|$, where $N > 0$.
14. Suppose the players of all six Smythe Division teams are randomly re-distributed for the 1991-92 season. How likely is at least one team expected to finish in the same position in 1991-92 as in the 1990-91 season? Determine your answer to within 1% accuracy. Repeat the problem for all 24 NHL teams common to the two seasons.